Class note Math 152H-1 August 28

1. Reviewed the Syllabus. Download it!

2. Limits of sequences

   (a) A **sequence** is an infinite list of numbers. For example, $1, \frac{1}{2}, \frac{1}{3}, \ldots$. We will write these as $\{a_n\}_{n=1}^{\infty}$ where a_n is the n^{th} number in the list. $a_3 = \frac{1}{3}$ in the sequence above. The sequence is said to **converge** if a_n gets closer and closer to a *single, finite* number, L, as n gets larger and larger. This number is called the **limit**. The limit of the sequence above is 0. Limits are very common in mathematics, and are the basis for calculus. For a converging sequence we write $\lim_{n\to\infty} a_n = L$.

   (b) Not all sequences converge. These are said to **diverge** and the limit is said not to exist. Some like $1, 2, 3, \ldots$ get bigger and bigger. Some get more or more negative, these are said to diverge to infinity (positive or negative). Some divergent sequences just bounce between numbers, like $1, -1, 1, -1, \ldots$.

   (c) The way we've written the examples so far hides an error. We don't actually know what happens after we write $\ldots$ In our cases the pattern is pretty clear. However, we don't actually *know* that this is the pattern. In fact, what we are asking in finding the limit is pretty difficult: this is an infinite list. How do we know the terms (the $1,000,000^{th}$ say) in a way that will allow us to find the limit. The answer is that we will give a rule for finding the terms one by one. For our first example above, we use

   $$a_n = \frac{1}{n}$$

   then we can find the millionth term by setting $n = 1,000,000$.

3. Examples : Say whether the following converge or diverge. If they converge, to what do they converge? Remember the key is to think about what happens as $n \to \infty$?

   $$\lim_{n\to\infty} \tfrac{1}{2^n} \qquad \lim_{n\to\infty} (-1)^n\left(2 - \tfrac{1}{n^2}\right)$$

   $$\lim_{n\to\infty} \tfrac{3n+1}{2n+1} \qquad \lim_{n\to\infty} n - \tfrac{1}{n}$$

   Ans: 0, diverges, converges to $\frac{3}{2}$, diverges to $+\infty$.

   In class we also looked at $\lim_{n\to\infty} (-1)^n n$ and $\lim_{n\to\infty} (-1)^n \frac{1}{n}$. These along with the second sequence above give three possibilities: the one above is trying to converge to both 2 and -2 (hence no limit) since $2 - \frac{1}{n^2}$ tends to 2. $\lim_{n\to\infty} (-1)^n n$ also alternates between negative and positive numbers, but now n goes to ∞ so there is no limit (and because of the positive and negative, does not diverge to either $+\infty$ or $-\infty$). The last converges to 0 even though there is negative and positive alternation.

   We also calculated the following limits:

   $$\lim_{n\to\infty} \frac{3n+1}{2n+1} = \lim_{n\to\infty} \frac{\frac{3n+1}{n}}{\frac{2n+1}{n}} = \lim_{n\to\infty} \frac{3 + \frac{1}{n}}{2 + \frac{1}{n}} = \frac{3}{2}$$

   and

   $$\lim_{n\to\infty} \frac{1 - 2n^2}{\sqrt{n} - 1} = \lim_{n\to\infty} \frac{\frac{1-2n^2}{\sqrt{n}}}{\frac{\sqrt{n}-1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}} - 2n^{\frac{3}{2}}}{1 - \frac{1}{\sqrt{n}}} = -\infty$$

   The trick is to divide top and bottom by the lowest power of n, having converted all square roots, etc. into powers of n. We will use powers of n (or x) all the time, so if you have problems with the above computations look at appendix 9 in the book (which reviews formulas from algebra, geometry and trigonometry).