

The test will cover sections 12.2-12.5, 13.1, 13.3, and 13.4.

1) Given $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{w} = -\vec{i} + \vec{j} + \vec{k}$, you should be able to do the following:

1. Find $\|\vec{v}\|$ and the direction of \vec{v} .
2. Find $\vec{v} \bullet \vec{w}$ and $\vec{v} \times \vec{w}$.
3. Use the computations in 1) to find the angle between these vectors and the equation of the plane containing them.
4. Find the equation of a plane with normal \vec{w} through the point $(2, 1, -1)$.
5. Find the vector $\text{proj}_{\vec{v}}\vec{w}$

2) Use projection to find the distance from

- a) The point $(1, -1, 2)$ to the line $\vec{r} + t\vec{w}$ where $\vec{r} = -\vec{j}$ and $\vec{w} = -\vec{i} + \vec{j} + 2\vec{k}$.
- b) The point $(1, -1, 2)$ and the plane $2x - 3y + 4z = 2$.

3) Find the values of s , $0 \leq s \leq 2\pi$ for which the tangent line to

$$\vec{r}(s) = \cos(s)\vec{i} - \sin(2s)\vec{j} + s\vec{k}$$

is parallel to the xz -plane. Find a parameterization of the tangent line to this curve at $s = \pi$. Where does this tangent line intersect $x - y + z = 3$?

4) You are given that

$$\frac{d\vec{r}}{dt} = (t^2 - 1)\vec{i} + e^t\vec{j} - t^2\vec{k}, \quad \vec{r}(0) = \vec{k}$$

find $\vec{r}(3)$.

5) Consider the following two parameterizations:

$$\begin{aligned}\vec{r}(t) &= t\vec{i} + 4t^{\frac{3}{2}}\vec{j} + 2t^{\frac{3}{2}}\vec{k} \\ \vec{w}(t) &= \sin^2 3t\vec{i} + (4|\sin^3 3t|)\vec{j} + (2|\sin^3 3t|)\vec{k}\end{aligned}$$

Find the length of the path swept out by $\vec{r}(t)$ for $0 \leq t \leq 1$. I contend that you can then tell me the length of the path swept out by $\vec{w}(t)$ for $0 \leq t \leq \pi$ without any additional integrals. How is this possible? What is that length? (recall $\sqrt{x^2} = |x|$)

6) Find the unit tangent for $\vec{r}(t) = 2t^4\vec{i} + t^4\vec{j} - 5t^4\vec{k}$. What kind of curve does this parameterize?

7) Find the unit tangent, $\vec{T}(t)$, the unit principal normal $\vec{N}(t)$, and the curvature $\kappa(t)$ for each value of t for the curve:

$$\vec{r}(t) = \frac{t^3}{3}\vec{i} + \frac{t^2}{2}\vec{j}$$

for $t > 0$. More such problems can be found on pg 942. The odd numbered ones have answers in the back of the book.