

The test will cover sections 12.2-12.5, 13.1, 13.3, and 13.4.

1) Given  $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$  and  $\vec{w} = -\vec{i} + \vec{j} + \vec{k}$ , you should be able to do the following:

1. Find  $||\vec{v}||$  and the direction of  $\vec{v}$ .
2. Find  $\vec{v} \bullet \vec{w}$  and  $\vec{v} \times \vec{w}$ .
3. Use the computations in 1) to find the angle between these vectors and the equation of the plane containing them.
4. Find the equation of a plane with normal  $\vec{w}$  through the point  $(2, 1, -1)$ .
5. Find the vector  $proj_{\vec{v}}\vec{w}$

2) Use projection to find the distance from

- a) The point  $(1, -1, 2)$  to the line  $\vec{r} = t\vec{w}$  where  $\vec{r} = -\vec{j}$  and  $\vec{w} = -\vec{i} + \vec{j} + 2\vec{k}$ .
- b) The point  $(1, -1, 2)$  and the plane  $2x - 3y + 4z = 2$ .

3) Find the values of  $s$ ,  $0 \leq s \leq 2\pi$  for which the tangent line to

$$\vec{r}(s) = \cos(s)\vec{i} - \sin(2s)\vec{j} + s\vec{k}$$

is parallel to the  $xz$ -plane. Find a parameterization of the tangent line to this curve at  $s = \pi$ . Where does this tangent line intersect  $x - y + z = 3$ ?

4) You are given that

$$\frac{d\vec{r}}{dt} = (t^2 - 1)\vec{i} + e^t\vec{j} - t^2\vec{k}, \quad \vec{r}(0) = \vec{k}$$

find  $\vec{r}(3)$ .

5) Consider the following two parameterizations:

$$\vec{r}(t) = t\vec{i} + 4t^{\frac{3}{2}}\vec{j} + 2t^{\frac{3}{2}}\vec{k}$$

$$\vec{w}(t) = \sin^2 3t\vec{i} + (4|\sin^3 3t|)\vec{j} + (2|\sin^3 3t|)\vec{k}$$

Find the length of the path swept out by  $\vec{r}(t)$  for  $0 \leq t \leq 1$ . I contend that you can then tell me the length of the path swept out by  $\vec{w}(t)$  for  $0 \leq t \leq \pi$  without any additional integrals. How is this possible? What is that length? (recall  $\sqrt{x^2} = |x|$ )

6) Find the unit tangent for  $\vec{r}(t) = 2t^4\vec{i} + t^4\vec{j} - 5t^4\vec{k}$ . What kind of curve does this parameterize?

7) Find the unit tangent,  $\vec{T}(t)$ , the unit principal normal  $\vec{N}(t)$ , and the curvature  $\kappa(t)$  for each value of  $t$  for the curve:

$$\vec{r}(t) = \frac{t^3}{3}\vec{i} + \frac{t^2}{2}\vec{j}$$

for  $t > 0$ . More such problems can be found on pg 942. The odd numbered ones have answers in the back of the book.