

For more on partial derivatives, see section 14.3.

1) Calculate the following partial derivatives:

$$\begin{aligned} \frac{\partial}{\partial x} (3x - y + 4) & \quad \frac{\partial}{\partial y} (3x - y + 4) \\ \frac{\partial}{\partial x} \left(\frac{x^2}{y} + \ln(1 + x^2 y^2) \right) & \quad \frac{\partial}{\partial y} (\cos(y e^{-xy})) \\ \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} (x^3 y + xy) \right) & \quad \frac{\partial}{\partial y} \left(\frac{\partial}{\partial x} (x^3 y + xy) \right) \end{aligned}$$

2) Find the tangent plane to

$$g(x, y) = xy^2 + \sin(\pi\sqrt{x})$$

at the point $(4, 1)$. Use this to approximate $g(1.01, 1.99)$

3) Find $\frac{\partial}{\partial x} f$ and $\frac{\partial}{\partial y} f$ at $(0, 0)$ where

$$f(x, y) = \begin{cases} 0 & |y| \geq x^2 \\ (x^2 - y)^2 & |y| \leq x^2 \end{cases}$$

4) You are given a function $w = h(x, y, z)$. Explain, using the definitions, what $\frac{\partial h}{\partial z}$ should mean at $(1, 2, -1)$. (For instance what slope?).

5) Suppose that $f(x, y)$ is differentiable at (a, b) . If $f(a, b) \geq f(x, y)$ (local max) for all (x, y) in an open neighborhood of (a, b) , explain why the both partial derivatives of $f(x, y)$ must vanish at (a, b) .