See sections 14.1 and 14.2 for definitions. There are graphs of functions where the limit does not exist at $(0,0)$, similar to the example given in class. See pgs 979-980 (there are some more on pg 982, corresponding to exercises 35, 36).

1) Explain why $\mathbb{R}^2 \backslash \{(0,0)\}$ (all points in $\mathbb{R}^2$ except the origin) is an open subset of $\mathbb{R}^2$.

2) Calculate the following limits:

$$\lim_{(x,y)->(2,-3)} ln(1+x^2y^2)$$

$$\lim_{(x,y)->(0,0),\ xy\neq 0} \frac{sin(xy)}{xy}$$

$$\lim_{(x,y)->(2,2),\ x+y\neq 4} \frac{x+y-4}{\sqrt{x+y}-2}$$ (hint: difference of squares)

$$\lim_{(x,y)->(0,0)} x\,cos\,\frac{1}{y}$$ (hint: squeeze theorem, a.k.a. sandwich theorem)

3) Show that

$$\lim_{\vec{x}\to\vec{0}} g(\vec{x})$$

does not exist when

$$g(x,y,z) = \frac{xy^2z}{x^4+y^4+z^4}$$

(hint: how does one describe a line through the origin in $\mathbb{R}^3$?).

4) Let

$$h(x,y) = \begin{cases} 0 & y > x^2, y < \frac{x^2}{3} \\ 1 & \frac{x^2}{3} \leq y \leq x^2 \end{cases}$$

What is this function's domain? Show that $\lim_{x\to 0} h(x,kx) = 0$ for any $k \in \mathbb{R}$. Thus, regardless of the line you use to come into the origin, you will get 0 as the limit. However, find a path on a parabola for which the limit will be 1, thereby showing that $\lim_{(x,y)\to(0,0)} h(x,y)$ does not exist.