Two more questions about lines and planes and then some about the lengths of curves (See sections 12.5, 13.3 for examples, but if you don't want it given away you should wait until you're stuck):

1) Find the intersection between the line parameterized as $(1-s)\,\vec{i}-2s\,\vec{j}+(3+s)\,\vec{k}$ and the plane $2\,x-3\,y+4\,z=2$. If we have a line $\vec{r}_0+t\,\vec{w}$ and a plane $(\vec{p}-\vec{p}_0)\bullet\vec{N}=0$ (where $\vec{p}=x\,\vec{i}+y\,\vec{j}+z\,\vec{k}$ and $\vec{p}_0$ is a vector to the point in the plane) is there a way to determine if the line intersects the plane or not (using vectors, dot and cross products, etc). When does the line lie in the plane? When is the line perpendicular to the plane?

2) You are given two planes:

$$\begin{aligned}2x-y-2z&=-2\\x-y+z&=0\end{aligned}$$

Find normals to each of these planes. Find the angle between these normals (assuming that they are based at the same point). This is defined to be the angle between the planes assuming that they intersect. Find the line of intersection between the two planes (note: you should get a parameterized line as in the previous problem)(hint: this line must be perpendicular to both normals).

3) Find the distance a particle has travelled from time $t=0$ until time $t=1$ if its path is parameterized by:

a) $\vec{r}(t)=3\,sin\,t\,\vec{i}+3\,cos\,t\,\vec{j}+4t\,\vec{k}$

b) $\vec{r}=t^3\,\vec{i}-2t^3\,\vec{j}-3t^3\,\vec{k}$

4) Consider $\vec{r}(t)=e^t\,cos\,t\,\vec{i}+e^t\,sin\,t\,\vec{j}+e^t\,\vec{k}$. Find $\vec{r}(0)$. If this parameterizes the path of a particle, how far has the particle travelled starting at time 0 and ending at time 3? How about at any time, t? If $s(t)$ is this length, find a formula for $s(t)$, and the parameterization by arc-length $\vec{r}(s)$.

(Challenge Problem) Let $\vec{r}(s)$ be the parameterization of a curve γ, where s is the arc-length from $\vec{r}(0)$. We saw in class that the following equations hold (I have been more explicit about what depends on s)

$$\begin{aligned}\frac{d\vec{T}(s)}{ds}&=\kappa(s)\,\vec{N}(s)\\\frac{d\vec{N}(s)}{ds}&=\tau(s)\,\vec{B}(s)-\kappa(s)\,\vec{T}(s)\\\frac{d\vec{B}(s)}{ds}&=-\tau(s)\vec{N}(s)\end{aligned}$$

Here are some questions to think about for Monday:

a) What types of curves have $\kappa(s)=0$ for all s?

b) What types of curves have $\tau(s)=0$ for all s?

c) What types of curves have $\tau(s)=0$ and $\kappa(s)=1$ for all s?

(Hint: Think geometrically about what these conditions mean for the derivatives of $\vec{T},\vec{B}$)