1) Find the angle between the vectors $\vec{v} = 1\,\vec{i} + \sqrt{2}\,\vec{j} + -\sqrt{2}\,\vec{k}$ and $\vec{w} = -1\,\vec{i} + 1\,\vec{j} + 1\,\vec{k}$.

2) Write $-\vec{i} + 2\,\vec{k}$ as a sum of its projection in the direction of $\vec{w} = \vec{i} + \vec{j} + \vec{k}$ and a vector perpendicular to $\vec{w}$.

3) Verify that the following two vectors, in $\mathbb{R}^2$, are perpendicular unit vectors:

$$\begin{aligned} \vec{u}_1 &= \tfrac{1}{2}\vec{i} + \tfrac{\sqrt{3}}{2}\vec{j} \\ \vec{u}_2 &= -\tfrac{\sqrt{3}}{2}\vec{i} + \tfrac{1}{2}\vec{j} \end{aligned}$$

Find $a$ and $b$ such that $\vec{i} = a\,\vec{u}_1 + b\,\vec{u}_2$.

4) Show that $(\lambda\vec{v}_1 + \vec{v}_2) \bullet \vec{w} = \lambda(\vec{v}_1 \bullet \vec{w}) + (\vec{v}_2 \bullet \vec{w})$ and $\vec{v} \bullet \vec{w} = \vec{w} \bullet \vec{v}$.

5) Let $\vec{v}(t)$ and $\vec{w}(t)$ be two differentiable vector-valued functions. Then the dot product $\vec{v}(t) \bullet \vec{w}(t)$ is a function of $t$. Write down a formula for

$$\frac{d}{dx}\left(\vec{v}(t) \bullet \vec{w}(t)\right)$$

using only vectors and their derivatives (first use components, but you need to find a way to suppress them in the end). If $\vec{v}(t)$ has length 1 for every value of $t$, show that $\vec{v}(t) \bullet \frac{d\vec{v}}{dt} = 0$ for every value of $t$ (hint: differentiate $||v(t)||^2$)