1) Given

$$\vec{r}(t) = cos\, t\, \vec{i} + sin\, t\, \vec{j} + sin\, 2t\, \vec{k}$$

find

a) $\dfrac{d\vec{r}}{dt}$.

b) The velocity vector at $t = \frac{\pi}{2}$.

c) The direction (unit tangent vector) of the velocity at $t = \frac{\pi}{2}$.

d) A parameterization of the tangent line to this curve at $t = \frac{\pi}{2}$.

e) Approximate $\vec{r}(\frac{\pi}{2} - 0.001)$.

2) Given

$$\vec{w}(s) = cos(s^3 + 1)\, \vec{i} + sin(s^3 + 1)\, \vec{j} + sin(2 + 2s^3)\, \vec{k}$$

Explain the relationship between this vector valued function and the one in the previous problem. Use this to find $\frac{d\vec{w}}{ds}$. What value of s corresponds to the point $t = \frac{\pi}{2}$ in the previous problem? Verify that the unit tangent vectors at this point are the same for the two problems.

3) Find a parameterization $\vec{r}(t)$ satisfying

$$\frac{d\vec{r}}{dt} = e^{2t}\, \vec{i} + (3t^2 + sin\, \pi t)\, \vec{j} + \frac{1}{t}\, \vec{k} \qquad \vec{r}(1) = \vec{j} - \vec{k}$$

To do this, rewrite the derivative as in $\vec{i}, \vec{j}, \vec{k}$ notation and break the problem into three separate differential equations.

4) (More challenging!) Explain what the following means, using Riemann sums:

$$\int_a^b \vec{v}(t)\, dt$$

How would we compute this direct integral if $\vec{v}(t) = v_1(t)\, \vec{i} + v_2(t)\, \vec{j} + v_3(t)\, \vec{k}$?