

1) Find the area of the portion of the plane $z = -x$ inside $x^2 + y^2 = 9$

2) Compute the following surface integral

$$\iint_S x \, d\sigma$$

where S is $\{(x, y, z) | y = x^2, 0 \leq x \leq 2, 0 \leq z \leq 3\}$.

3) Use the divergence theorem to compute the flux of $\vec{F} = x^2 \vec{i} + xz \vec{j} + 3z \vec{k}$ out of solid sphere $x^2 + y^2 + z^2 \leq 4$.

4) Let $\vec{F} = x \vec{i} + y \vec{j} + z \vec{k}$, and suppose that the surface S bounds a region V to which the divergence theorem applies. Calculate

$$\iint_S \vec{F} \bullet \vec{n} \, d\sigma$$

where \vec{n} is the outward pointing normal to S .

5) Let $f(x, y, z)$ be a function on a region V such that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

1. Calculate $\nabla \bullet (\nabla f)$ and $\nabla \bullet (f \nabla f)$ in terms of f and ∇f .

2. Compute the flux of ∇f for a surface S , bounding a region V , to which the divergence theorem applies.

3. Show that

$$\iint_S (f \nabla f) \bullet \vec{n} \, d\sigma = \iiint_V \|\nabla f\|^2 \, dV$$