For Wednesday, read section 16.6

1) Calculate the (outward) flux and (counter-clockwise) circulation for the vector field $\vec{F} = (x+y)\vec{\imath} - (x^2+y^2)\vec{\jmath}$ on the triangle formed by $y = 0$, $x = 1$, and $y = x$.

2) Suppose $f(x, y)$ is a function on $\mathbb{R}^2$ with continuous partial derivatives of all orders. Furthermore, for each pair of a simple closed curve C and a region $\mathcal{R}$ bounded by C, we have

$$\oint_C \frac{\partial f}{\partial y}\, dx - \frac{\partial f}{\partial x}\, dy = 0$$

In class, we saw that this is minus the flux integral for ∇f. Compute $\nabla \bullet (\nabla f)$ by using the expression for $\nabla \bullet \vec{F}$ as a limit of flux integrals given at the end of class. Write out the result in terms of partial derivatives (instead of ∇'s).

3) Suppose $\vec{F}(x, y)$ is the velocity vector field for a planar fluid flow on a simply connected region $\mathcal{R}$. A streamline for $\vec{F}$ is the path of an individual particle in the fluid (i.e. it's tangent vector at $\vec{u}$ is $\vec{F}(\vec{u})$). Suppose that $\nabla \bullet \vec{F} > 0$ at every point in $\mathcal{R}$. Show that there is no closed streamline in $\mathcal{R}$ (hint: what can you compute for closed curves in $\mathcal{R}$?). This is called *Bendixson's criterion). Interpret the result by explaining what the condition* $\nabla \bullet \vec{F} > 0$ *says at each point of* $\mathcal{R}$