Read section 16.4, especially the sections on flux and circulation.

1) Compute the following line integral using Green's theorem:

$$\oint_C xy^2\,dx + (x^2y + 2x)\,dy$$

where C is $(x-2)^2 + (y-3)^2 = 4$.

2) Compute the following line integrals:

$$\oint_C 4x^3y\,dx + x^4\,dy \qquad\qquad \oint_C f(x)\,dx + g(y)\,dy$$

where C is any closed curve that bounds a region $\mathcal{R}$, to which Green's theorem applies. What is special about the vector fields defining these line integrals?

3) Explain why

$$area(\mathcal{R}) = \frac{1}{2}\oint_C -y\,dx + x\,dy$$

when $\mathcal{R}, C$ can be used in Green's theorem. Use this to compute the area of the region enclosed by

$$\vec{r}(t) = t^2\,\vec{i} + (\frac{t^3}{3} - t)\,\vec{j} \qquad\qquad -\sqrt{3} \le t \le \sqrt{3}$$

(you might try graphing C!)

4) Among all simple, closed curves in the plane that are oriented counterclockwise, find the one along which

$$\oint_C \vec{F} \bullet \vec{T}\,ds$$

is maximal, when $\vec{F}(x,y) = \left(\frac{1}{4}x^2y + \frac{1}{3}y^3\right)\vec{i} + x\,\vec{j}$. (Hint: where is the double integral in Green's theorem positive?)

5) Let $f(x,y)$ be a function which has continuous partial derivatives of every order. If

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

on $\mathbb{R}^2$, show that

$$\oint_C \frac{\partial f}{\partial y}\,dx - \frac{\partial f}{\partial x}\,dy = 0$$

for all curves C to which Green's theorem applies. The first equation is known as the Laplace equations. It shows up in physics and engineering a lot; for example, it describes the conditions for steady state for heat flows. We'll see how to interpret the second equation on Monday.