1) Compute the following

$$\int_\gamma \sqrt{x}y\,dx + \frac{x}{z}\,dy + \frac{1}{1+z^2}\,dz$$

for γ parameterized as $\vec{r}(t) = t^2\,\vec{i} + t^3\,\vec{j} + t\,\vec{k}$, $0 \leq t \leq 1$.

2) Which of the following is a conservative vector field?

a) $\vec{F}(x,y,z) = z^2\,ln\,y\,\vec{i} + \dfrac{xz^2}{y}\,\vec{j} + 2xz\,ln\,y\,\vec{k}$

b) $\vec{F}(x,y,z) = x^2yz\,\vec{i} + \dfrac{1}{3}x^3y^2z\,\vec{j} + sin\,z\,\vec{k}$

Compute $\int_\gamma \vec{F} \bullet \vec{r}$ for γ given by $\vec{r}(t) = t^2\,\vec{i} + e^{-t^2}\,\vec{j} + sin(cos(t))\,\vec{k}$, $0 \leq t \leq 2$, for the conservative vector field F.

3) Compute the following line integrals for the vector field $\vec{F}$ around/across the square with vertices $(-1,-1)$, $(1,-1)$, $(1,1)$, $(-1,1)$ (in that order):

$$\vec{F}(x,y) = x^2\,\vec{i} + y^2\,\vec{j}$$

and

$$\oint_\gamma \vec{F} \bullet \vec{T}\,ds$$

$$\oint_\gamma \vec{F} \bullet \vec{n}\,ds$$

where $\vec{T}$ is the unit tangent vector and $\vec{n}$ is the unit vector normal to the path and pointing out of the square. The first integral is called the circulation and the second is called the flux. We'll talk more about these later.

4) Compute the curl and divergence of the following vector field at $(0,0,2)$.

$$\vec{F}(x,y,z) = xy\,\vec{i} + yz\,\vec{j} + xyz\,\vec{k}$$

5) Which of the following sets is simply connected?

$$\{(x,y,z)|1 \leq z^2 + y^2 \leq 2\}$$

$$\{(x,y,z)|1 \leq z^2 + y^2 + z^2 \leq 2\}$$

Find the value of the following line integral

$$\int_\gamma \frac{1}{x^2+y^2+z^2}(x\,dx + y\,dy + z\,dz)$$

over *any* path γ from $(0,0,-1)$ to $(0,0,1)$ and lying on $x^2+y^2+z^2=1$.