For more about vector fields: see section 16.2, for more about path independence, exact differentials, see 16.3

1) For each of the following differentials on $\mathbb{R}^2$, find which is exact, and if it is exact, find a function $F(x, y)$ such that $\frac{\partial F}{\partial x} = P(x, y)$ and $\frac{\partial F}{\partial y} = Q(x, y)$.

a) $(x^3 + y^3)\, dx + 3xy^2\, dy$

b) $-y^2\, dx + x^2\, dy$

c) $(\frac{3}{2}\sqrt{x}y^2 + x\, sin\, y)\, dx + (2x^{\frac{3}{2}}y + \frac{1}{2}x^2\, cos\, y)\, dy$

2) For a) and b) in problem #1, find the line integral over line segment from $(2, 0)$ to $(0, 2)$.

3) Let $f(x)$ be a continuous function of x. Let γ be the closed path formed by completing the three segments $(1, f(1)) \to (1, 0) \to (3, 0) \to (3, f(3))$ by travelling from $(3, f(3))$ to $(1, f(1))$ along the graph of f. Evaluate the following circuit integral

$$\oint_\gamma -y\, dx$$

in terms of $f(x)$.

4) We considered the following circuit integral in class:

$$\oint_\gamma \frac{-y}{x^2 + y^2}\, dx + \frac{x}{x^2 + y^2}\, dy$$

We showed that when γ is the unit circle, this integral equals 2π, and that the differential satisfied $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. However, since the domain of the differential is not simply connected, this did not imply the differential is exact. Compute this integral for γ equal to the circle of radius $\frac{3}{2}$, centered at $(1, 0)$. (Hint: find simply connected regions inside the domain of the differential which allow you to use path independence and the calculation on the unit circle to find the answer. Do not, unless you really like doing work, find a parameterization and plug in!).