See sections 16.1 for more about line integrals.

1) Compute the following integral

$$\int_\gamma x^3y + y^3x\,ds$$

along the straight line from $(0,0)$ to $(1,2)$. Use the parameterization by arc length.

2) Calculate the following line integrals:

a) $\displaystyle\int_\gamma 3y\,dx + 4x\,dy$ along the arc parameterized by $\gamma(t) = t^2\,\vec{i} + 3\sqrt{t}\,\vec{j}$, $0 \le t \le 2$

b) $\displaystyle\int_\gamma sin\,x\,dx + cos\,y\,dy$ along the curve formed by the horizontal line segment from $(0,1)$ to $(2,1)$ followed by the vertical line segment from $(2,1)$ to $(2,0)$ (You will need to use two integrals for the two parts).

3) Let $d\sigma = \frac{\partial F}{\partial x}\,dx + \frac{\partial F}{\partial y}\,dy$ at all points in an open set $\mathcal{R}$. Let γ_1 and γ_2 be two parameterized paths beginning and ending at the same points, and contained wholly in $\mathcal{R}$. What can you say about

$$\int_{\gamma_1} d\sigma \qquad \int_{\gamma_2} d\sigma$$

(recall the special way to evaluate line integrals in this case)

4) Find $F(x,y)$ such that

$$\frac{\partial F}{\partial x} = 3x^2y \qquad \frac{\partial F}{\partial y} = x^3 + sin\,y$$

and use this to evaluate

$$\int_\gamma 3x^2y\,dx + (x^3 + sin\,y)\,dy$$

over the path parameterized by $\vec{r}(t) = (2\sqrt{t+1})\,\vec{i} + \pi(t^2+1)\,\vec{j}$, $0 \le t \le 3$.

5) Let $\gamma(t) = (x(t), y(t))$, $0 \le t \le 1$ be a parametrization of a curve γ defined inside the domain of $P(x,y)$ and $Q(x,y)$. Then $\overline{\gamma}(t) = \gamma(1-t)$ parameterizes the same curve, but switches the starting point and the end point. Show that

$$\int_{\overline{\gamma(t)}} P\,dx + Q\,dy = -\int_{\gamma(t)} P\,dx + Q\,dy$$

(evaluate the integral on the left using the new parameterization)