In all cases, draw a sketch of the region of integration!.

1) Re-write

$$\int_0^1 \int_{-x^2}^0 \int_0^{y^2} f(x, y, z)\, dz\, dy\, dx$$

as an integral using the orders $dy\, dz\, dx$ and $dx\, dz\, dy$.

2) Compute

a)

$$\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dx\, dy\, dz$$

b)

$$\int_0^\pi \int_0^\pi \int_0^\pi cos(u + v + w)\, du\, dv\, dw$$

3) Find the volume of the region cut from the cylinder $x^2 + y^2 = 4$ by the planes $z = 0$ and $x + z = 3$.

4) Compute the Jacobian $\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)}$ for the spherical coordinates change of variables:

$$\begin{aligned} x &= \rho\, sin\, \phi\, cos\, \theta \\ y &= \rho\, sin\, \phi\, sin\, \theta \\ z &= \rho\, cos\, \phi \end{aligned}$$

and verify that it equals $\rho^2 sin\, \phi$.

5) Find the volume of the region above $z = x^2 + y^2$ and below $x^2 + y^2 + z^2 = 2$ by evaluating a triple integral in *cylindrical coordiantes*.

6) Describe the region, $\mathcal{R}$, where $x^2 + y^2 + z^2 \leq 4$, $z \geq 0$, and $3z^2 \leq x^2 + y^2$ in terms of spherical coordinates. Use this to compute the integral

$$\int\int\int_{\mathcal{R}} \frac{3}{1 + (x^2 + y^2 + z^2)^{\frac{3}{2}}}\, dV$$