To compute certain double integrals, it helps to convert to polar coordinates. Recall that we use the substitution $x = r\,cos\,\theta$ and $y = r\,sin\,\theta$. This allows us to think of x and y as functions of (r, θ). I said in class that

$$\int\int_{\mathcal{R}} f(x,y)dA = \int\int_{\mathcal{R}} f(r\,cos\,\theta, r\,sin\,\theta)\,r\,dr\,d\theta$$

where we will write the limits for $\mathcal{R}$ in terms of r and θ. See section 15.3

1) Verify that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$ where

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

What happens if we use $x = r\,sin\,\theta$, $y = r\,cos\,\theta$ instead?

2) What region of (x,y) pairs corresponds to $1 \le r \le 1.1$ and $0 \le \theta \le 0.1\pi$ under the maps $x = r\,cos\,\theta$ and $y = r\,sin\,\theta$. Compare the area of this (x,y)-region to the area of the region of (x,y) pairs corresponding to $100 \le r \le 100.1$ (which is also of length 0.1) and $\pi \le \theta \le 1.1\,\pi$ (the same θ width).

3) Compute the volume of the solid whose between $z = 0$ and $z = \dfrac{tan^{-1}(\frac{y}{x})}{\sqrt{x^2+y^2}}$ and sitting over the region $0 \le r \le 1 + \frac{1}{2}\,cos\,\theta$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$.

4) Find the area of the region enclosed by the positive x-axis and the points (r,θ) such that $r = \frac{4}{3}\theta$, $0 \le \theta \le \frac{\pi}{2}$ (hint: draw a picture, and recall that a double integral of the function $F(x,y) = 1$ gives the area of the region of integration).

5) Change the integral to polar coordinates and compute its value:

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$$

6) Compute the following integral

$$\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$