1) You are given a function $f(x, y)$ whose domain is $\mathbb{R}^2$. You wish to calculate $\int\int_{\mathcal{R}} f(x, y)\, dA$. Find the limits in the integrals for each of the following regions

a) $\mathcal{R} = \{(x, y)|0 \leq x \leq 2, x^3 \leq y \leq 4x$ and we want the limits in $\int_?^? \int_?^? f(x, y)dy\, dx$ (note the order of integration)

b) $\mathcal{R}$ is the bounded triangular region containing $(1, 0)$ whose boundary is on the lines $y = -x$, $y = 2x$ and $y = 3x - 4$. Find the limits for $\int_?^? \int_?^? f(x, y)dx\, dy$.

2) Compute the following:

a) The integral of $f(x, y) = x^2 + y^2$ over the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.

b) The volume of the solid which is beneath $z = x + 4$ and above the bounded region between $y = 4 - x^2$ and $y = 3x$, an

3) Compute the following integral by changing the order of integration:

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx$$

4) Let $a, b, c \geq 0$ be four numbers. The planes $x = 0$, $y = 0$, $z = 0$, and $a\, x + b\, y + z = c$ cut out a pyramid one of whose vertices is $(0, 0, 0)$. Evaluate a double integral to show that the volume of this pyramid is $\frac{1}{3}$ the are of the its base times its height.

5) Compute $\int_0^2 tan^{-1}(\pi\, x) - tan^{-1}(x)\, dx$. (Hint: make use of a substitution of the form $tan^{-1}(\pi\, x) - tan^{-1}(x) = \int_1^{\pi} ?\, dy$ and change the order of integration.)