1) A farmer wants to build a tank holding 8000 ft^3 volume. The tank should be a cylinder with hemispherical ends. What radius and height of the cylinder will minimize the total surface area of the tank? (and hence the cost). Use Lagrangian multipliers to solve this.

2) Find the point closest to the origin on the line of intersection of the planes $y + 2z = 12$ and $x + y = 6$. (Note: the minimum of the distance from the origin to (x, y, z) occurs when $x^2 + y^2 + z^2$ is minimized!).

3) Which of the following limits exist and why:

$$\lim_{(x,y)\to(1,0)} \frac{xy - y}{y^2 - y}$$

$$\lim_{(x,y)\to(1,0)} \frac{xy - y}{x^2 - 2\,x + 1 + y^2}$$

4) Describe the level sets of the function $G(x, y) = \sqrt{x^2 - y}$ (You don't need to take any derivatives!).

5) Show that if $w = f(s)$ is a differentiable function, and $s = y + 5x$, then

$$\frac{\partial w}{\partial x} - 5\frac{\partial w}{\partial y} = 0$$

6) Let $f(x, y, z) = ln(2x + 3y + 6z)$ and $P = (-1, -1, 1)$. What is the domain of f? Describe this domain. What is ∇f? What is the rate of change of f in the direction of $\vec{v} = 2\,\vec{i} + 3\,\vec{j} + 6\,\vec{k}$? In what direction does f decrease fastest at P? Find the equation of the tangent plane and normal line to the level set of f through P.

7) Find and classify the critical points of $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$. Try to describe the level sets of this function.

8) What is the equation for the tangent plane at to the graph of the function

$$f(x, y) = x^2 - xy + y^2 - 3$$

at the point $(1, 2)$. Is f more sensistive to small changes in x or y at the point $(1, 2)$?

9) Suppose the surfaces $f(x, y, z) = 0$ and $g(x, y, z) = 0$ intersect in a curve, γ, through the point (a, b, c). Assuming that f and g are differentiable everywhere, find a parameterization for the tangent line to γ at this point. When does such a tangent line not exist?

10) The equations $e^u cos\, v = x$ and $e^u sin\, v = y$ define u and v in terms of x and y (this is not obvious). Show that the angle between

$$\frac{\partial u}{\partial x}\,\vec{i} + \frac{\partial u}{\partial y}\,\vec{j} \qquad\qquad \frac{\partial v}{\partial x}\,\vec{i} + \frac{\partial v}{\partial y}\,\vec{j}$$

is the same at every point.