The chain rule(s) are covered in section 14.5.

1) Find the following derivatives, in terms of s, t.

a) $\dfrac{\partial w}{\partial s}$ where $w = xy + sin\, z$ and $x = s^2 + t$, $y = st$ and $z = ln\, s$.

b) $\dfrac{d\,z}{d\,t}$ where $z = xy + yz$ and $x(t) = t$, $y(t) = t^2$, $z(t) = t^3$. Do this first by plugging in, and then by using the chain rule.

2) Let $T(x, y)$ be the temperature at each point (x, y). Suppose

$$\frac{\partial T}{\partial x} = y \qquad \frac{\partial T}{\partial y} = x$$

Find the locations of the maximum and minimum temperature on the ellipse $x = 2\sqrt{2}\, cos\, t$, $y = 2\sqrt{2}\, sin\, t$ (hint: use the chain rule to calculate the derivatives of T to find critical points, concavity, etc).

3) If $f(u, v, w)$ is differentiable and $u = x - y$, $v = y - z$, $w = z - x$, show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$$

(caution: note that the roles of u and x have been changed!).

Given $f(x, y)$, we can convert f to polar coordinates using

$$x = r\, cos\, \theta \qquad y = r\, sin\, \theta$$

Find $\dfrac{\partial\, f}{\partial\, \theta}$ and $\dfrac{\partial\, f}{\partial\, r}$ in terms of $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$. Interpret these in terms of directional derivatives of f.