1) Find the equation of the tangent line, and the normal line, to the curve

$$x^4y + y^2x = 6$$

at the point $(1, 2)$. (Hint: find a function $F(x, y)$ for which this curve is a level set).

2) For a function $h(x, y, z)$, ∇h at (a, b, c) will still be normal to the level set, $\{(x, y, x)|h(x, y, z) = C\}$ through (a, b, c). Given

$$h(x, y, z) = x^2 + 3y^2 + 6z^2$$

find a parameterization of the line normal to the level set through $(1, 1, 1)$. Find the equation of the plane tangent to this level set.

3) Use the gradient to draw a qualitative, but reasonably accurate, sketch of the level sets for

$$g(x, y) = xy$$

(It's useful to know what is happening at points of the forms $(0, y)$ and $(x, 0)$). Verify that this reflects the solutions of $xy = c$ for different values of c.

4) Do the same as in #3, but for the function

$$g(x, y) = x^3 - 3xy^2$$

This one is quite hard, and you won't be able to do the verification at the end. To get started, find the level set $g(x, y) = x(x^2 - 3y^2) = 0$.