1) Find the absolute maximum/minimum (if it exists) of the function

$$f(x, y) = x^2 + 2y^2 - 2x + 12y$$

on the set $-\infty < x \leq 0, 0 \leq y < \infty$. Remember that it may not occur at a critical point.

2) Compute ∇f for the function

$$f(x, y) = x^3 y + sin(xy)$$

at the point $(1, \pi)$. In what direction from $(1, \pi)$ does the function *decrease* fastest?

3) Perhaps unsurprisingly, ∇h for $h(x, y, z)$ is defined to be

$$\nabla h = \frac{\partial h}{\partial x}\,\vec{i} + \frac{\partial h}{\partial y}\,\vec{j} + \frac{\partial h}{\partial z}\,\vec{k}$$

Compute ∇h for $h(x, y, z) = x^2 y + 3z + 5$.

4) Show that the linearization of $F(x, y)$ or $F(x, y, z)$ at a point $\vec{p}$ (assume it is differentiable there) can be written as

$$z = f(\vec{p}) + \nabla f\big|_{\vec{p}} \cdot (\vec{r} - \vec{p})$$

where $\vec{r} = x\,\vec{i} + y\,\vec{j}$ ($+z\,\vec{k}$ if we have three variables).

5) To compute the concavity of $f(x, y)$ at a point (a, b) in the direction determined by a unit vector, $\vec{u}$, first find $D_{\vec{u}}(f)$ as a function of (x, y). Since this is a function of (x, y) we can compute the "second derivative in the direction $\vec{u}$" by $D_{\vec{u}}(D_{\vec{u}} f)$. Use this to compute:

a) the concavities of $g(x, y) = xy$ in the directions $\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ and $-\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$,

b) the concavities of $h(x, y) = 2x^2 + 3xy + y^2$ in the directions $\vec{i}$ and $\frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}$.