The second derivative test is summarized on pg 1033. Section 14.7 is about finding max/min/saddles for two variable functions.

1) Show that $(0,0)$ is a critical point of

$$z = 2x^2 + 3xy + y^2$$

Describe what sort of point it is (local max/min, saddle, or none of the above) and use this if possible to describe what a diagram of the level sets would look like (you won't be able to pin down the diagram exactly, so describe it qualitatively).

2) Find the critical points for $f(x,y) = 3x^2y + y^3$, and classify them as a max/min, saddle, or neither.

3) Find and classify the critical points on $x > 0, y > 0$ for $f(x,y) = xy + 2x - ln(x^2y)$.

4) Consider, for $a, b, c \in \mathbb{R}$ the function

$$f(x,y) = a\,x^2 + 2b\,xy + c\,y^2$$

Find the critical points for this function. Show that

$$D_{(0,0)}(f) = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 4ac - 4b^2$$

Rewrite the function, by completing the square, as

$$a(x + ?)^2 + ?\,y^2$$

(you need to fill in the question marks). Use this to relate the sign of $D_{(0,0)}(f)$ to the level sets for c slightly bigger and slightly smaller than $f(0,0) = 0$.