SOLUTIONS TO REVIEW PROBLEMS

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1. $\lim_{x\to 3}\dfrac{9-x^2}{x^2-x-6}=\lim_{x\to 3}\dfrac{(3-x)(3+x)}{(x-3)(x+2)}=\lim_{x\to 3}\dfrac{-(3+x)}{x+2}=-\dfrac{6}{5}$

2. $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to 4}\dfrac{(x-4)(\sqrt{x}+2)}{x-4}$

$=\lim_{x\to 4}(\sqrt{x}+2)=2+2=4$

3. $\lim_{x\to 1^-}\dfrac{2x+1}{x^2-1}=\dfrac{3}{0^-}=-\infty$

4. $\lim_{x\to 0}\dfrac{3x}{\tan 4x}=\lim_{x\to 0}\dfrac{3x}{\frac{\sin 4x}{\cos 4x}}=\lim_{x\to 0}\dfrac{3x}{\sin 4x}\cdot\cos 4x=\lim_{x\to 0}\dfrac{3}{4}\cdot\dfrac{4x}{\sin 4x}\cdot\cos 4x$

$=\dfrac{3}{4}\cdot 1\cdot 1=\dfrac{3}{4}$

5. $\lim_{t\to 0}\dfrac{\sin 2t}{\sin 3t}=\lim_{t\to 0}\dfrac{\sin 2t}{2t}\cdot\dfrac{3t}{\sin 3t}\cdot\dfrac{2t}{3t}=1\cdot 1\cdot\dfrac{2}{3}=\dfrac{2}{3}$

6. $\lim_{\theta\to 0}\cos\left(\dfrac{\pi\theta}{\sin\theta}\right)=\cos\left(\lim_{\theta\to 0}\pi\cdot\dfrac{\theta}{\sin\theta}\right)=\cos(\pi\cdot 1)=\cos(\pi)=-1$

7. <u>First Method</u>: $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{2\theta}=\lim_{\theta\to 0}\dfrac{1-\cos\theta}{2\theta}\cdot\dfrac{1+\cos\theta}{1+\cos\theta}$

$=\lim_{\theta\to 0}\dfrac{1-\cos^2\theta}{2\theta(1+\cos\theta)}=\lim_{\theta\to 0}\dfrac{\sin^2\theta}{2\theta(1+\cos\theta)}=\lim_{\theta\to 0}\dfrac{1}{2}\cdot\dfrac{\sin\theta}{\theta}\cdot\sin\theta\cdot\dfrac{1}{1+\cos\theta}$

$=\dfrac{1}{2}\cdot 1\cdot 0\cdot\dfrac{1}{2}=0$

<u>Second Method</u>: The half angle formula states that

$$1-\cos\theta=2\sin^2\!\left(\frac{\theta}{2}\right)$$

Hence $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{2\theta}=\lim_{\theta\to 0}\dfrac{\sin^2(\frac{\theta}{2})}{\theta}=\lim_{\theta\to 0}\dfrac{\sin(\frac{\theta}{2})}{\theta}\cdot\sin\left(\dfrac{\theta}{2}\right)$

$$= \lim_{\theta \to 0} \frac{\sin(\frac{\theta}{2})}{\frac{\theta}{2}} \cdot \frac{1}{2} \cdot \sin\left(\frac{\theta}{2}\right) = 1 \cdot \frac{1}{2} \cdot 0 = 0$$

8. <u>Main Idea</u>: Divide both top and bottom by the highest power of x in the denominator, in this case x^2.

$$\lim_{x \to \infty} \frac{2\sqrt{x} + x^{\frac{3}{2}}}{x^2 - 5} = \lim_{x \to \infty} \frac{\frac{2}{x^{3/2}} + \frac{1}{x^{1/2}}}{1 - \frac{5}{x^2}} = \frac{0 + 0}{1 - 0} = 0$$

9. <u>Main Idea</u>: Divide both top and bottom by the highest power of x in the denominator, in this case x^2.

$$\lim_{x \to -\infty} \frac{x^2 + x + 1}{2x^2 - x + 1} = \lim_{x \to -\infty} \frac{1 + \frac{1}{x} + \frac{1}{x^2}}{2 - \frac{1}{x} + \frac{1}{x^2}} = \frac{1 + 0 + 0}{2 - 0 + 0} = \frac{1}{2}$$

10. $f'(x) = \left(x - \frac{1}{x}\right) \cdot 3(3x^2 - 5x)^2(6x - 5) \; + \; (3x^2 - 5x)^3 \cdot \left(1 + \frac{1}{x^2}\right)$

11. $g'(x) = (5x\sqrt{1 + 2x})(\cos^2 x)' + (\cos^2 x)(5x\sqrt{1 + 2x})'$

$$= (5x\sqrt{1 + 2x}) 2\cos x(-\sin x) + (\cos^2 x)(5x \cdot \tfrac{1}{2}(1 + 2x)^{-\frac{1}{2}} \cdot 2 + \sqrt{1 + 2x} \cdot 5)$$

$$= -10x\sqrt{1 + 2x}\cos x \sin x + \frac{5x\cos^2 x}{\sqrt{1 + 2x}} + 5\cos^2 x\sqrt{1 + 2x}$$

12. Note that $h(x) = \left(\sin\left(\frac{x}{\cos x}\right)\right)^2$. Hence

$$h'(x) = 2\sin\left(\frac{x}{\cos x}\right)\left(\sin\left(\frac{x}{\cos x}\right)\right)' = 2\sin\left(\frac{x}{\cos x}\right)\cos\left(\frac{x}{\cos x}\right)\left(\frac{x}{\cos x}\right)'$$

$$= 2\sin\left(\frac{x}{\cos x}\right)\cos\left(\frac{x}{\cos x}\right)\frac{\cos x + x\sin x}{\cos^2 x}$$

13. $k'(x) = \dfrac{(x^2 + 1) - (x + 1)2x}{(x^2 + 1)^2} = \dfrac{1 - x^2 - 2x}{(x^2 + 1)^2}$

For Problems 14, 15 and 16 use the definition that

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

14. $f'(x) = \lim_{h \to 0} \dfrac{\sqrt{2(x + h) + 1} - \sqrt{2x + 1}}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{2x + 1 + 2h} - \sqrt{2x + 1}}{h} \cdot \frac{\sqrt{2x + 1 + 2h} + \sqrt{2x + 1}}{\sqrt{2x + 1 + 2h} + \sqrt{2x + 1}}$$

$$= \lim_{h\to 0} \frac{(2x+1+2h)-(2x+1)}{h(\sqrt{2x+1+2h}+\sqrt{2x+1})}$$
$$= \lim_{h\to 0} \frac{2h}{h(\sqrt{2x+1+2h}+\sqrt{2x+1})}$$
$$= \lim_{h\to 0} \frac{2}{\sqrt{2x+1+2h}+\sqrt{2x+1}} = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

15. $g'(x) = \lim_{h\to 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} = \lim_{h\to 0} \frac{\frac{(2x+1)-(2x+2h+1)}{(2x+2h+1)(2x+1)}}{h} = \lim_{h\to 0} \frac{\frac{-2h}{(2x+2h+1)(2x+1)}}{h}$
$$= \lim_{h\to 0} \frac{-2}{(2x+2h+1)(2x+1)} = \frac{-2}{(2x+1)^2}$$

16. $h'(x) = \lim_{h\to 0} \frac{(2(x+h)+1)^2-(2x+1)^2}{h}$
$$= \lim_{h\to 0} \frac{((2(x+h)+1)-(2x+1))\,((2(x+h)+1)+(2x+1))}{h}$$
$$= \lim_{h\to 0} \frac{(2h)(4x+2h+2)}{h} = \lim_{h\to 0} (2)(4x+2h+2)$$
$$= 2(4x+2) = 8x+4$$

17. $\frac{dy}{dx} = 2\sec x \cdot \sec x \tan x = 2\sec^2 x \tan x\,.$

When $x=0$, $y=\sec^2(0)=1$. Slope of the tangent line through $(0,1)$ is equal to $\left.\frac{dy}{dx}\right|_{x=0} = 2\sec^2(0)\tan(0) = 0\,.$

Equation of the tangent line: $(y-1)=0(x-0)$, i.e., $y=1\,.$

18. $x^2+xy+y^2=1$. Differentiating both sides with respect to x (implicit differentiation) we get,

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \quad\Longrightarrow\quad (x+2y)\frac{dy}{dx} = -2x-y$$

Hence $\frac{dy}{dx} = \frac{-2x-y}{x+2y}$, and the slope of the tangent line through $(1,-1)$ is

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{-2(1)-(-1)}{1+2(-1)} = 1\,.$$

Equation of the tangent line: $y-(-1)=1(x-1)$, i.e., $y=x-2\,.$

19. $y^2 = \frac{x-1}{x+1}$. Differentiating both sides with respect to x we obtain,

$$2y\frac{dy}{dx} = \frac{(x+1)(1)-(x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2} \quad\Longrightarrow\quad \frac{dy}{dx} = \frac{1}{y(x+1)^2}$$

Slope of the tangent through $(2, \frac{1}{\sqrt{3}})$ is $\frac{1}{\frac{1}{\sqrt{3}}(2+1)^2} = \frac{\sqrt{3}}{9}$.

Equation of the tangent line: $(y - \frac{1}{\sqrt{3}}) = \frac{\sqrt{3}}{9}(x-2)$.

20. $f(x) = \sqrt{5-x^2}$. Note that the domain of the function is $-\sqrt{5} \le x \le \sqrt{5}$. (Otherwise we get square root of a negative number!)

$$f'(x) = \frac{1}{2}(5-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{5-x^2}}$$

Critical points of $f(x)$: $f'(x) = 0$ implies $x = 0$. $f'(x)$ is not defined at $x = \sqrt{5}$ and $x = -\sqrt{5}$. But these last two are already end points and need not be considered as critical points.

Next evaluate $f(x)$ at the critical points and end points:

$$f(-\sqrt{5}) = 0, \qquad f(0) = \sqrt{5}, \qquad f(\sqrt{5}) = 0$$

Finally, comparing these values of f we conclude that f has one absolute maximum $(0, \sqrt{5})$ and two absolute minima $(-\sqrt{5}, 0)$ and $(\sqrt{5}, 0)$.

21. $g'(x) = -3 \cdot \dfrac{2}{3}x^{-\frac{1}{3}} = \dfrac{-2}{x^{\frac{1}{3}}}$

Critical points of $g(x)$: Since the numerator of $g'(x)$ is -2, $g'(x)$ is never zero. But $g'(x)$ is not defined for $x = 0$. So the only critical point of $g(x)$ is $x = 0$.

Next evaluate $g(x)$ at the critical point and end points:

$$g(-1) = -3(-1)^{\frac{2}{3}} = -3((-1)^2)^{\frac{1}{3}} = -3(1)^{\frac{1}{3}} = -3(1) = -3$$

Similarly, $g(0) = 0$ and $g(1) = -3$.

We conclude that g has one absolute maximum $(0,0)$ and two absolute minima $(-1,-3)$ and $(1,-3)$ in the given interval.

22. Let V be the volume of the water in the tank at time t. Let h be the height of the water at time t, and let r be the radius of the surface of the water at time t. Then $V = \frac{1}{3}\pi r^2 h$ and we are given that $\frac{dV}{dt} = 9$. We are asked to find $\frac{dh}{dt}\big|_{h=6}$. By similarity of right triangles it is easy to see that $\frac{h}{r} = \frac{10}{5} = 2$. Hence $r = \frac{1}{2}h$ and $V = \frac{\pi}{3} \cdot \frac{1}{4}h^3 = \frac{\pi}{12}h^3$. By the Chain Rule,

$$9 = \frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}.$$

When $h = 6$,

$$\left.\frac{dh}{dt}\right|_{h=6} = 9 \cdot \frac{4}{\pi h^2} = \frac{36}{\pi \cdot 36} = \frac{1}{\pi} \text{ ft/min}$$

23. $y' = \dfrac{2(x^2-4) - 2x(2x)}{(x^2-4)^2} = \dfrac{-8-2x^2}{(x^2-4)^2} = \dfrac{-2(x^2+4)}{(x^2-4)^2}$

Note that we always have $y' < 0$.

$$y'' = \frac{-4x(x^2-4)^2 + 2(x^2+4)\cdot 2(x^2-4)\cdot 2x}{(x^2-4)^4}$$

$$= \frac{-4x(x^2-4) + 8x(x^2+4)}{(x^2-4)^3} = \frac{4x^3+48x}{(x^2-4)^3} = \frac{4x(x^2+12)}{(x^2-4)^3}$$

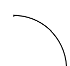
$y'' = 0 \implies x = 0$. Note that

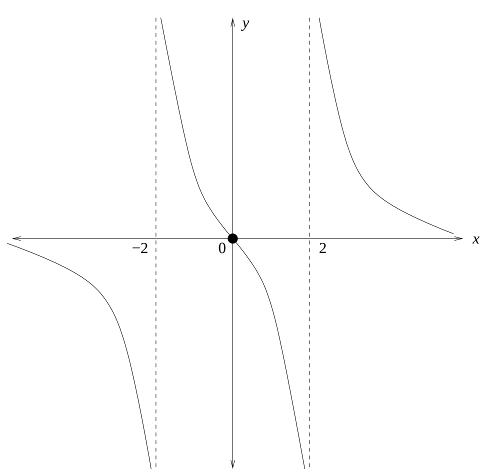
$$\text{sign}\,(y'') = \text{sign}\left(\frac{x}{x^2-4}\right) = \text{sign}\,(x(x^2-4)) = \text{sign}\,(x(x-2)(x+2)).$$

$x = \pm 2$ are vertical asymptotes. The x-axis is a horizontal asymptote since

$$\lim_{x\to\pm\infty} \frac{2x}{x^2-4} = \lim_{x\to\pm\infty} \frac{\frac{2}{x}}{1-\frac{4}{x^2}} = \frac{0}{1-0} = 0$$

Finally note that y is an odd function, and its graph passes through the origin $(0,0)$.

	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,2)$	2	$(2,\infty)$
sign(y')	$-$		$-$	$-$	$-$		$-$
sign(y'')	$-$		$+$	0	$-$		$+$
shape of y				0			



24. $y = x\sqrt{8-x^2}$ is an odd function. Note that we must have

$$8 - x^2 \geq 0 \quad \Longrightarrow \quad x^2 \leq 8 \quad \Longrightarrow \quad -\sqrt{8} \leq x \leq \sqrt{8}\,.$$

$$y' = \sqrt{8-x^2} + x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{8-x^2}} \cdot (-2x) = \sqrt{8-x^2} - \frac{x^2}{\sqrt{8-x^2}}$$

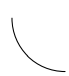
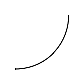
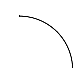
$$= \frac{8 - x^2 - x^2}{\sqrt{8-x^2}} = \frac{2(4-x^2)}{\sqrt{8-x^2}}$$

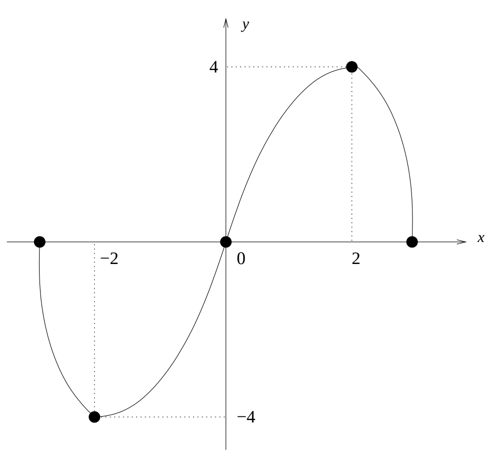
$y' = 0 \implies x = \pm 2\,,$ and $\operatorname{sign}(y') = \operatorname{sign}(4 - x^2)$.

$$y'' = \frac{-4x\sqrt{8-x^2} - 2(4-x^2)\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{8-x^2}}\cdot(-2x)}{8-x^2}$$

$$= \frac{-4x(8-x^2) + 2(4-x^2)x}{(8-x^2)\sqrt{8-x^2}} = \frac{2x^3 - 24x}{(8-x^2)\sqrt{8-x^2}} = \frac{2x(x^2-12)}{(8-x^2)^{3/2}}$$

$y'' = 0 \implies x = 0, \pm\sqrt{12}\,,$ but note that $x = \pm\sqrt{12}$ are outside the domain. Since we always have $8 - x^2 \geq 0\,,$ $x^2 - 12 < 0\,,$ it follows that $\operatorname{sign}(y'') = \operatorname{sign}(x(x^2-12)(8-x^2)) = \operatorname{sign}(-x)$.

	$-\sqrt{8}$	$(-\sqrt{8}, -2)$	-2	$(-2, 0)$	0	$(0, 2)$	2	$(2, \sqrt{8})$	$\sqrt{8}$
$\operatorname{sign}(y')$	$-\infty$	$-$	0	$+$	$+$	$+$	0	$-$	$-\infty$
$\operatorname{sign}(y'')$	$+\infty$	$+$	$+$	$+$	0	$-$	$-$	$-$	$-\infty$
shape of y	0		-4		0		4		0



25. $f'(x) = \dfrac{1}{2\sqrt{x}}$ and $f'(9) = \dfrac{1}{2\cdot 3} = \dfrac{1}{6}$.

$$L(x) = f'(9)(x-9) + f(9) = \frac{1}{6}(x-9) + 3 = \frac{x}{6} - \frac{3}{2} + 3 = \frac{x}{6} + \frac{3}{2}.$$

$$\sqrt{10} = f(10) \approx L(10) = \frac{10}{6} + \frac{3}{2} = \frac{10+9}{6} = \frac{19}{6}.$$

26. The approximate change in volume is $dV = 4\pi r_0^2\,dr$.

27. Let V be the volume and S be the surface area of the box. Let x be the length of the sides of the square base and y be the height of the box. Then $S = 2x^2 + 4xy$, and we are given that $V = x^2y = 32$, which implies that $y = \frac{32}{x^2}$. Hence we have

$$S = 2x^2 + 4x \cdot \frac{32}{x^2} = 2x^2 + \frac{128}{x}$$

$$\frac{dS}{dx} = 4x - \frac{128}{x^2} = \frac{4x^3 - 128}{x^2} = \frac{4(x^3-32)}{x^2}$$

$$\frac{dS}{dx} = 0 \quad \Longrightarrow \quad x^3 = 32 \quad \Longrightarrow \quad x = \sqrt[3]{32}$$

The only other critical point of S is $x = 0$, which is the zero of the denominator of $\frac{dS}{dx}$. Since $x > 0$, we can exclude this critical point. Since $\frac{dS}{dx} > 0$ to the right of $x = \sqrt[3]{32}$, and $\frac{dS}{dx} < 0$ to the left of $x = \sqrt[3]{32}$, the function S will have absolute minimum at $x = \sqrt[3]{32} = 2^{5/3}$. The corresponding y value is

$$y = \frac{32}{x^2} = \frac{32}{(\sqrt[3]{32})^2} = \sqrt[3]{32} = x$$

Hence the minimal surface area occurs when $x = y = \sqrt[3]{32}$

$$S_{\min} = 2x^2 + 4x^2 = 6x^2 = 6(\sqrt[3]{32})^2 = 6(2^{\frac{5}{3}})^2 = 6\cdot 2^{\frac{10}{3}} = 6\cdot 2^{3+\frac{1}{3}} = 6\cdot 2^3\cdot 2^{\frac{1}{3}} = 48\sqrt[3]{2}$$

28. The constraint is $x^2 + y^2 = 5$, which implies that $y = \sqrt{5 - x^2}$. Therefore $s = 2x + \sqrt{5-x^2}$, and

$$\frac{ds}{dx} = 2 + \frac{1}{2\sqrt{5-x^2}} \cdot (-2x) = 2 - \frac{x}{\sqrt{5-x^2}} = \frac{2\sqrt{5-x^2} - x}{\sqrt{5-x^2}}$$

Since $0 < x < \sqrt{5}$,

$$\frac{ds}{dx} = 0 \quad \Longrightarrow \quad 2\sqrt{5-x^2} = x \quad \Longrightarrow \quad 4(5-x^2) = x^2$$

$$\Longrightarrow \quad 20 = 5x^2 \quad \Longrightarrow \quad x^2 = 4 \quad \Longrightarrow \quad x = 2$$

Note that $x = 2$ is the unique critical point of s when $0 < x < \sqrt{5}$, and one can check that the sign of $\frac{ds}{dx}$ changes from positive to negative at $x = 2$. Hence the absolute maximum of s is

$$s_{\max} = 2x + \sqrt{5 - x^2} = 2 \cdot 2 + \sqrt{5 - 4} = 4 + 1 = 5$$

29. Let $u = 7 - 3x^2$, then $du = -6x dx$ and

$$\int \frac{3x}{\sqrt{7 - 3x^2}}\, dx = \int -\frac{1}{2} \frac{du}{\sqrt{u}} = -\frac{1}{2} \int u^{-\frac{1}{2}}\, du = -\frac{1}{2} \cdot 2u^{\frac{1}{2}} + C = -\sqrt{7 - 3x^2} + C$$

30. Let $u = 2 + \sqrt{x}$, then $du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx$ and

$$\int \frac{(2 + \sqrt{x})^{\frac{3}{2}}}{\sqrt{x}}\, dx = \int u^{\frac{3}{2}} \cdot 2du = 2 \cdot \frac{2}{5} u^{\frac{5}{2}} + C = \frac{4}{5}(2 + \sqrt{x})^{\frac{5}{2}} + C$$

31. Let $u = 3x$, then $du = 3dx$ and

$$\int \sec 3x\, \tan 3x\, dx = \int \sec u\, \tan u \cdot \frac{1}{3} du = \frac{1}{3} \int \sec u\, \tan u\, du$$
$$= \frac{1}{3} \sec u + C = \frac{1}{3} \sec 3x + C$$

32. Let $u = \sin(x^2)$, then $du = \cos(x^2) \cdot 2x\, dx$ and

$$\int x\, \sin(x^2)\, \cos(x^2)\, dx = \int u \cdot \frac{1}{2} du = \frac{1}{2} \cdot \frac{u^2}{2} + C = \frac{1}{4} \sin^2(x^2) + C$$

33. $$\left(\frac{x}{2} - \frac{1}{2} \sin x\, \cos x + C\right)' = \frac{1}{2} - \frac{1}{2}(\sin x \cos x)'$$
$$= \frac{1}{2} - \frac{1}{2}(\cos x \cdot \cos x + \sin x(-\sin x)) = \frac{1}{2} - \frac{1}{2}(\cos^2 x - \sin^2 x)$$
$$= \frac{1}{2} - \frac{1}{2}(1 - \sin^2 x - \sin^2 x) = \sin^2 x$$

Hence the formula is correct.

34. Let $u = x + 1$, then $du = dx$, and
$x = 0 \Longrightarrow u = 1$, $\quad x = -1 \Longrightarrow u = 0$. Hence

$$\int_{-1}^{0} \sqrt{x+1}\, dx = \int_0^1 \sqrt{u}\, du = \frac{2}{3} u^{\frac{3}{2}} \Big]_0^1 = \frac{2}{3}$$

35. Let $u = 4 + 3\tan x$, then $du = 3\sec^2 x\, dx$, and
$x = \frac{\pi}{4} \Longrightarrow u = 4 + 3\tan\frac{\pi}{4} = 4 + 3 \cdot 1 = 7$,
$x = 0 \Longrightarrow u = 4 + 3\tan 0 = 4 + 3 \cdot 0 = 4$. Hence

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{(4 + 3\tan x)^2}\, dx = \int_4^7 \frac{1}{u^2} \cdot \frac{1}{3} du = \frac{1}{3} \cdot \frac{u^{-1}}{-1}\Big]_4^7 = -\frac{1}{3}\left(\frac{1}{7} - \frac{1}{4}\right)$$

$$= -\frac{1}{3} \cdot \frac{4-7}{28} = \frac{1}{28}$$

36. $\displaystyle\int_1^4 \frac{\sqrt{x}+x^2}{x}\,dx = \int_1^4 \left(\frac{1}{\sqrt{x}} + x\right) dx = \left[2 \cdot x^{\frac{1}{2}} + \frac{x^2}{2}\right]_1^4$

$$= \left(2\sqrt{4} + \frac{16}{2}\right) - \left(2 + \frac{1}{2}\right) = (4+8) - \left(2 + \frac{1}{2}\right) = 10 - \frac{1}{2} = \frac{19}{2}$$

37. The boundary curves are $y = x^2 - 2x$ and $y = x$. To find the x-coordinates of the intersection points, solve

$$x^2 - 2x = x \implies x^2 - 3x = 0 \implies x(x-3) = 0 \implies x = 0, 3$$

Note that the line $y = x$ lies above $y = x^2 - 2x$ over the interval $[0, 3]$. Hence the area is equal to

$$\int_0^3 (x - (x^2 - 2x))dx = \int_0^3 (-x^2 + 3x)dx = -\frac{x^3}{3} + \frac{3}{2}x^2\Big]_0^3 = -9 + \frac{3}{2} \cdot 9 = \frac{9}{2}$$

38. To find the y-coordinates of the intersection points, solve

$$y^2 = 3 \implies y = \pm\sqrt{3}$$

Note that the vertical line $x = 3$ lies to the right of the parabola $x = y^2$ when $-\sqrt{3} \le y \le \sqrt{3}$. Hence the area is equal to

$$\int_{-\sqrt{3}}^{\sqrt{3}} (3 - y^2)dy = \left[3y - \frac{y^3}{3}\right]_{-\sqrt{3}}^{\sqrt{3}} = \left(3\sqrt{3} - \frac{3\sqrt{3}}{3}\right) - \left(-3\sqrt{3} - \frac{-3\sqrt{3}}{3}\right)$$

$$= 2\left(3\sqrt{3} - \frac{3\sqrt{3}}{3}\right) = 2(3\sqrt{3} - \sqrt{3}) = 4\sqrt{3}$$

39. To find the x-coordinates of the intersection points, solve

$$x^2 = 4x - x^2 \implies 2x^2 - 4x = 0 \implies 2x(x-2) = 0 \implies x = 0, 2$$

Note that $y = 4x - x^2$ lies above $y = x^2$ over the interval $[0, 2]$. Hence the area is equal to

$$\int_0^2 ((4x - x^2) - x^2)dx = \int_0^2 (4x - 2x^2)dx = 2x^2 - \frac{2}{3}x^3\Big]_0^2$$

$$= 2 \cdot 4 - \frac{2}{3} \cdot 8 = \frac{1}{3} \cdot 8 = \frac{8}{3}$$

40. By the Fundamental Theorem of Calculus (Part 1), we have

$$\frac{d}{dx}\left(\int_2^x \sin^3\left(\frac{1}{t}\right) dt\right) = \sin^3\left(\frac{1}{x}\right)$$

41. Let $G(x) = \displaystyle\int_{-1}^x t\sqrt{1+t}\,dt$. Then $G'(x) = x\sqrt{1+x}$.

$$\frac{d}{dx}\left(\int_{-1}^{\sin x} t\sqrt{1+t}\,dt\right) = \frac{d}{dx}\left(G(\sin x)\right) = G'(\sin x)\cdot\cos x$$

$$= \sin x\,\sqrt{1+\sin x}\cdot\cos x = \sin x\,\cos x\,\sqrt{1+\sin x}$$

42. $y' = \int y''dx = \int (2x+1)\,dx = x^2 + x + C$

$2 = y'(0) = 0 + 0 + C = C$, and hence $y' = x^2 + x + 2$.

$$y = \int y'dx = \int (x^2 + x + 2)\,dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + D$$

$3 = y(0) = 0 + 0 + 0 + D = D$, and hence $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} + 2x + 3$.