Change-of-Base Formula.

For any logarithmic bases $a$ and $b$, and any positive number $M$,

$$\log_b M = \frac{\log_a M}{\log_a b}$$

**Problem #1.**

Use your calculator to find the following logarithms. Show your work with Change-of-Base Formula.

a) $\log_2 10$  

b) $\log_{\frac{3}{2}} 9$  

c) $\log_7 11$

Using the Change-of-Base Formula, we can graph Logarithmic Functions with an arbitrary base.  

*Example:*

$$\log_2 x = \frac{\ln x}{\ln 2}$$

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- Properties of Logarithms.

If $b$, $M$, and $N$ are positive real numbers, $b \neq 1$, $p$, $x$ are real numbers, then

1. $\log_b MN = \log_b M + \log_b N$  product rule

2. $\log_b \frac{M}{N} = \log_b M - \log_b N$  quotient rule

3. $\log_b M^p = p \log_b M$  power rule

4. \[
\begin{align*}
\log_b b^x &= x \\
\log_b x &= x, \quad x > 0
\end{align*}
\]  inverse property of logarithms

5. $\log_b M = \log_b N$ if and only if $M = N$.
   This property is the base for solving Logarithmic Equations in form $\log_b g(x) = \log_b h(x)$.

Properties 1-3 may be used for Expanding and Condensing Logarithmic expressions.

- Expanding and Condensing Logarithmic expressions.
Problem #2.

Express each of the following expressions as a single logarithm whose coefficient is equal to 1.

a) \( \frac{1}{5} \left[ 3 \log(x + 1) + 2 \log(x - 3) - \log 7 \right] \)

b) \( \frac{1}{2} \left[ \ln(x + 1) + 2 \ln(x - 1) \right] + \frac{1}{3} \ln x \)

c) \( \frac{1}{2} \ln(x + 3) - \frac{1}{5} \left[ \ln x + 3 \ln(x + 1) \right] \)

d) \( \frac{1}{2} \left[ \log(x - 2) + 2 \log(x + 2) - \log 5 \right] \)

Problem #3.

Expand as much as possible each of the following.

a) \( \log \frac{x^2 y}{z^5} \)

b) \( \ln \sqrt[4]{\frac{x^3 y}{z^3}} \)

- Solving Logarithmic Equations.
1. Solving the Simplest Logarithmic Equation (SLE).
Given: \( \log_b x = a, \ b > 0, \ b \neq 1, \ a \) is any real number.
According the definition of the logarithm this equation is equivalent to \( x = b^a \).

2. According to properties of logarithms, if \( \log_b M = \log_b N \), then \( M = N \).

Remember, check is part of solution for Logarithmic Equations.

**Problem #4.** Solve the following Logarithmic Equations.

a) \( \log_2 x = 5 \)

b) \( \log_3 (x - 2) = 5 \)

c) \( \log (x^2 - x) = \log 6 \)

d) \( \log_{\frac{1}{2}} (x + 4) = -3 \)
e) \( \log(x - 15) = -2 \)

f) \( \ln(x + 3) = 1 \)

g) \( \log(2x - 1) = \log(x - 2) \)