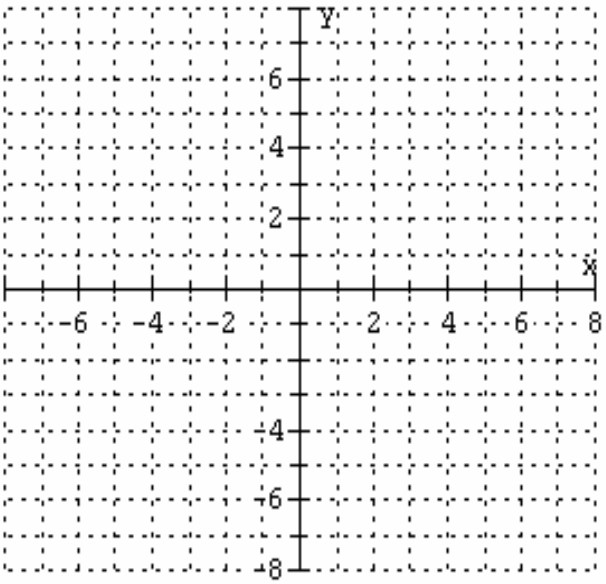
❖ General and Standard forms of the equation of Quadratic Functions.

Definition.
$f(x) = ax^2 + bx + c$, *where* a, b, c *are real numbers,* $a \neq 0$ is called quadratic function (or second degree polynomial function) in general form.
$f(x) = a(x-h)^2 + k$ is a standard form of the equation of a quadratic function, (h,k) is the vertex.

- Graphs of Quadratic Functions.

Problem #1.

Indicate how the graph of $f(x) = (x-3)^2 - 2$ is related to the graph of basic function $f(x) = x^2$. Sketch the graph of $y = (x-3)^2 - 2$. State the Range of f.

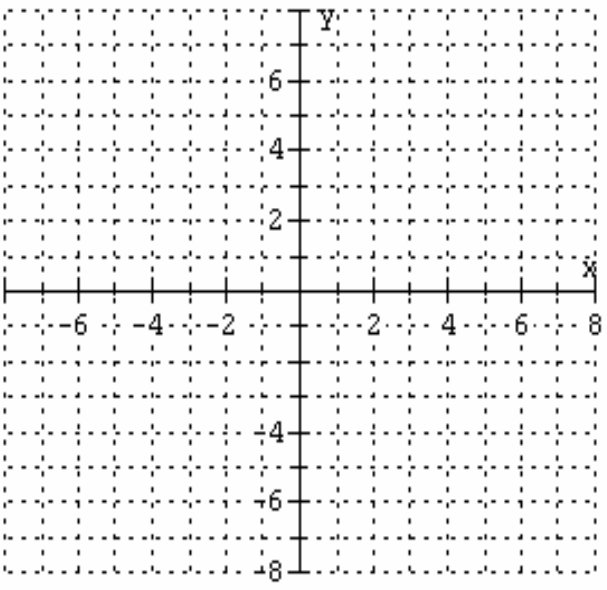


Problem #2.

Indicate how the graph of $f(x) = -(x-3)^2 - 2$ is related to the graph of the basic function $f(x) = x^2$.

Sketch the graph of $y = -(x-3)^2 - 2$.

State the Range of f.



Question. When a quadratic function has a maximum value? Minimum value?

- Properties of a Quadratic Function.
  $f(x) = ax^2 + bx + c, \quad a \neq 0$ - general form
  $f(x) = a(x-h)^2 + k$ - standard form, (h,k) is vertex.

1. Domain: $(-\infty, \infty)$

2. Vertex is the highest point on the parabola if $a < 0$ (quadratic function has a max value),
   and it is the lowest point on the parabola if $a < 0$ (quadratic function has a min value),

3. Range: $(-\infty, y_{vertex}]$ if $a < 0$; $[y_{vertex}, \infty)$ if $a > 0$.

4. Quadratic function increases on one of the vertex and decreases on the other.

5. Axis of symmetry: $x = h$ (parallel to x-axis).

6. Intercepts.

Problem #3.

For $f(x)=2\left(x-\frac{3}{4}\right)^2-8$ find the following:

a) Vertex.

b) Equation of the axis of symmetry.

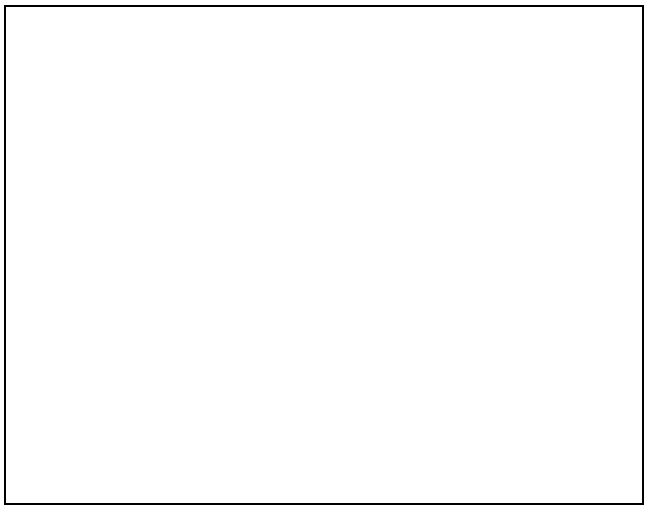
c) y- intercept.

d) x- intercepts.

e) Maximum/minimum.

f) Range.

g) Using the info above sketch the graph of $f(x)$



- Conversion general form of a quadratic function into standard form (completing square procedure).

Problem #4.

Convert the function $f(x) = -2x^2 + 5x - 5$ into standard form $f(x) = a(x-h)^2 + k$.

- Coordinates of the vertex when a quadratic function is given by an equation in general form.

For $f(x) = ax^2 + bx + c, \quad a \neq 0,$
the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a} \right) \right)$

Problem #5.

Find the vertex for $y = -3x^2 + 6x - 5$.

State the Range for $y = f(x)$.

- Applications.

Optimization.

Verbal problems involving optimization require to model functions based on given or implied conditions.
If the resulting function is quadratic, we can use the *x*-coordinate of the vertex to determine the maximum or minimum value of the function.

Note: When *x*-coordinate of the vertex is used to determine maximum or minimum value of a quadratic function, you need to **present a reason,** why given function has a maximum (minimum) value.

As the ***reason can be used info about the coefficient*** a in the equation of the quadratic function.

If the coefficient $a > 0$, parabola opens upward, the function has a minimum value.

If the coefficient $a < 0$, parabola opens downward, the function has a maximum value.

- Enclosing the Most Area with a Fence.

Problem #6.

A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along highway, what is the largest area that can be enclosed?

Problem #7.

A rancher has 600 yards of fencing to put around a rectangular field and then subdivide the field into two identical plots by placing a fence parallel to field's shorter sides. Find the dimensions that maximize enclosed area. What is the maximum area?

Problem #8.

Shannise Cole makes and sell candy. She has found that the cost per box for making *x* boxes of candy is given by

$$C(x) = x^2 - 40x + 405.$$

a) How much does it cost per box to make 15 boxes?

b) What point on the graph corresponds to the number of boxes that will make the cost per box as small as possible?

c) How many boxes should she make in order to keep the cost per box at a minimum?
What is the minimum cost per box?

Problem #9.

A bullet is fired upward from the ground level.
Its height above the ground (in feet) at time t seconds is given by

$$H = -16t^2 + 100t$$

Find the maximum height of the bullet and the time at which it hits the ground.

Problem #10.

The number of women employed full-time in civilian jobs has increased dramatically in the past century. The following table shows the number of employed women (in millions) in selected years.

Year	1910	1930	1940	1950	1960	1970	1980	1990	2000
Working Women	7.4	10.8	12.8	18.6	23.6	31.5	45.5	56.8	68.6

a) Display this info graphically.

b) Find a quadratic regression model for the data.

c) Use the regression model to estimate the number of women in the workforce in 2009.