- Reciprocal Function (the most basic rational function).

**Definition.**

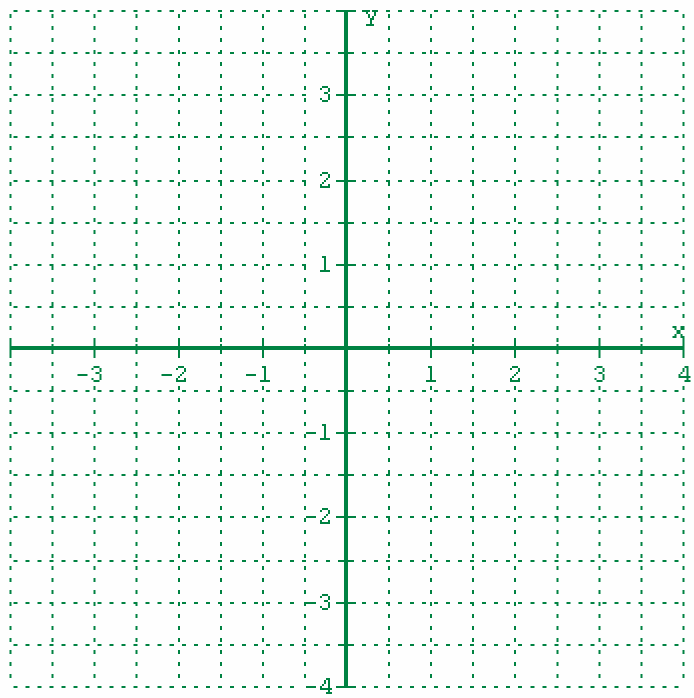
The Reciprocal Function is a function defined by

$$f(x) = \frac{1}{x}$$

**Problem #1.** Find the domain of the reciprocal function.

**Problem #2.** Sketch the graph of the reciprocal function by plotting points (using the table of function's values).

| $x$ | $-3$ | $-2$ | $-1$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | | | |




- Symbolic Description of the "end behavior" of $f(x) = \dfrac{1}{x}$.

| |
|---|
| As $x \to -\infty$, $f(x) \to 0$ and<br>as $x \to \infty$, $f(x) \to 0$ |

The line $y = 0$ ($x$ –axis) is said to be a ***horizontal asymptote*** of the graph of the reciprocal function. The curve approaches the *x*-axis.

- Definition of a Horizontal Asymptote.

> The line $y = b$ is a ***horizontal asymptote*** of the graph of a function $f$ if $f(x)$ approaches $b$ as $x$ increases or decreases without bound.

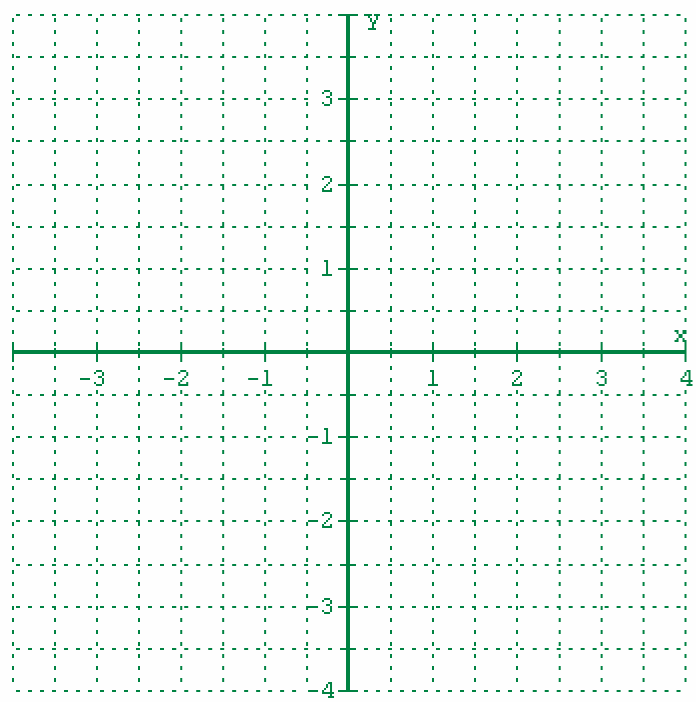
- Fill out the table for $x$ values close to 0.

| $x$ | $-0.5$ | $-0.1$ | $-0.01$ | $-0.001$ |
|---|---|---|---|---|
| $f(x)$ | | | | |

| $x$ | 0.5 | 0.1 | 0.01 | 0.001 |
|---|---|---|---|---|
| $f(x)$ | | | | |

**Problem #4.**

Make a sketch of the function $f(x) = \dfrac{1}{x}$ near 0.



- Symbolic Description of the behavior of $f(x) = \dfrac{1}{x}$ near 0.

> As $x \to 0^+$, $f(x) \to \infty$ and
> as $x \to 0^-$, $f(x) \to -\infty$

The line $x = 0$ ($y$ –axis) is said to be a ***vertical asymptote*** of the graph of the reciprocal function. The curve approaches, but does not touch, the *y*-axis.

- Definition of a Vertical Asymptote.

> The line $x = a$ is a ***vertical asymptote*** of the graph of a function $f$ if $f(x)$ increases or decreases without bound as $x$ approaches $a$.

- Range of the Reciprocal function.

- Transformations of the reciprocal functions and graphs.

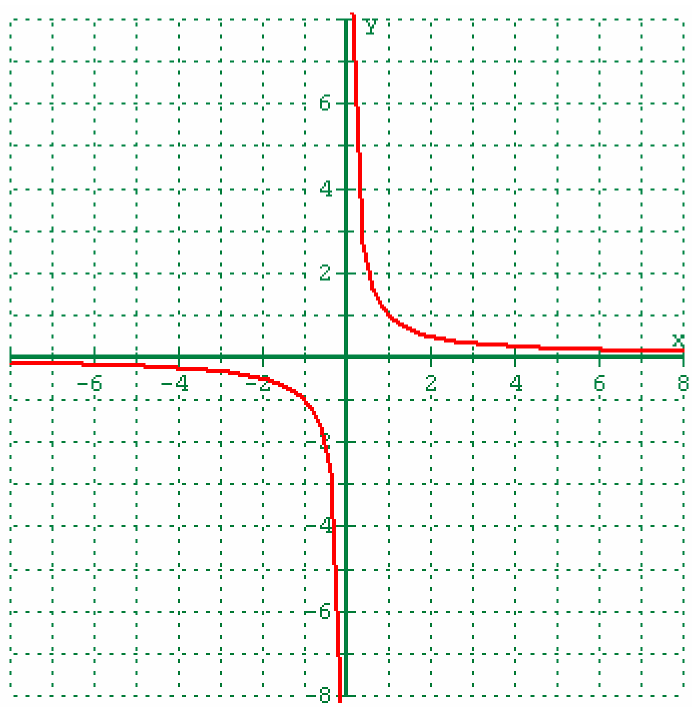
**Problem #5.**

a) Make a sketch of the graph of $g(x)=\dfrac{1}{x-3}$.

b) Write the equations of the vertical asymptote.

c) State the Domain of $g$.

Graph of the basic reciprocal function is provided.



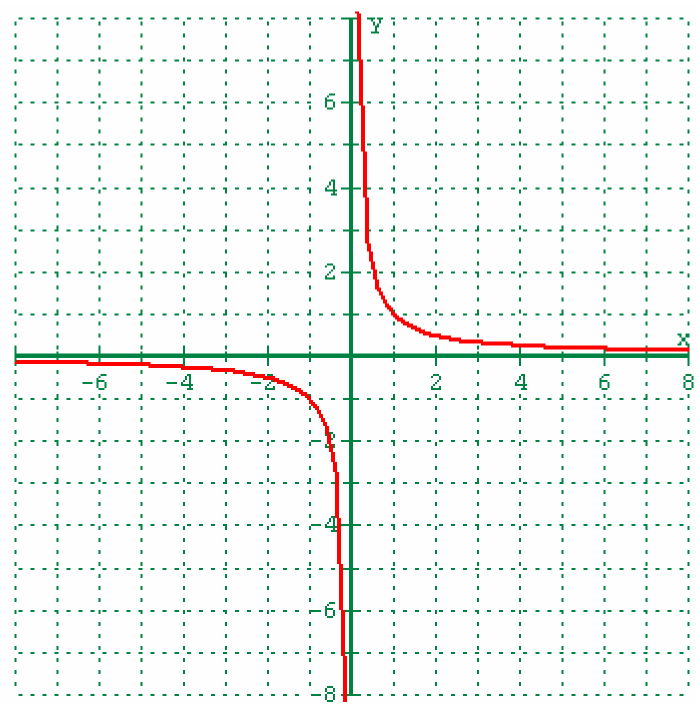
## **Problem #6.**

a) Make a sketch of the graph of $g(x) = \dfrac{1}{x} + 2$.

b) Write the equations of the vertical asymptote.

c) State the Domain of $g$.

Graph of the basic reciprocal function is provided.

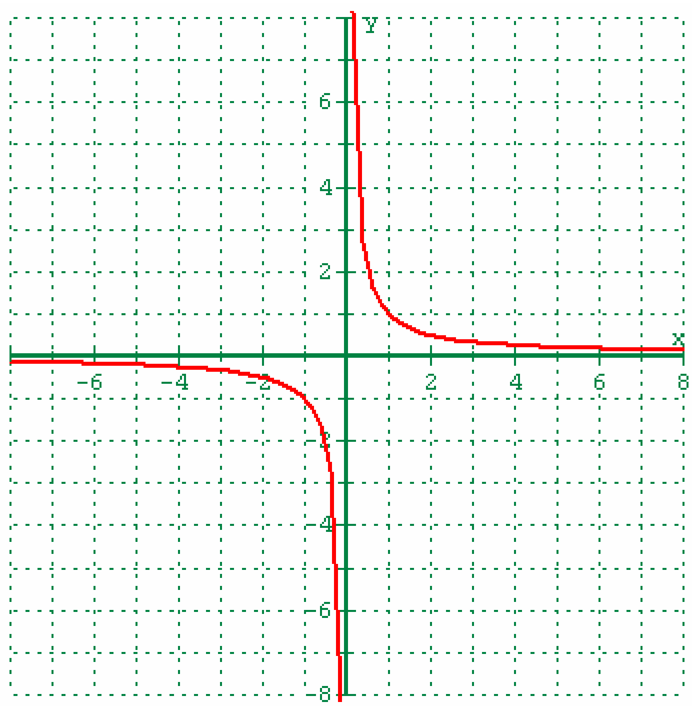



## **Problem #7.**

a) Write a sequence of transformations that leads from $f(x)=\dfrac{1}{x}$, to $h(x)=-\dfrac{1}{x+2}-4$.

b) Find the domain of *h*.

c) Sketch the graph of *h*. Show all intermediate graphs.



- Transformations of the basic reciprocal function belong to the class of Rational Functions.

- Definition of Rational Functions.

Rational Functions are quotients of two polynomials:

$$f(x)=\frac{p(x)}{q(x)},\quad q(x)\neq 0.$$

- Domain of Rational Functions. Vertical asymptotes.

The ***domain of a rational function*** is the set of all real numbers except the x-values that make the denominator zero.

If $x=x_0$ is a zero of the denominator of a rational function $f$, then $x=x_0$ is the equation for a vertical asymptote of the graph of $f$.

**Problem #8.**

(That problem can be used as a supplementary material).

Find the domain of the following rational functions. Write your answers using the interval notation. Write the equations of all vertical asymptotes for each function.

1. $f(x)=\dfrac{x-1}{x^2-2x+1}$

2. $g(x)=\dfrac{x+5}{x^2+x+1}$

3. $h(x)=\dfrac{2x}{x^2-2x-3}$

4. $k(x)=\dfrac{8}{x\left(x^2+3x-10\right)}$

5. $m(x)=\dfrac{10}{6x^2-17x+5}$

6. $s(x)=\dfrac{3(x-5)}{2x^2-x-1}$

- Modeling using variation.

Quantities can vary ***directly, inversely, or jointly***. We will consider only direct and inverse variation.

- Direct variation.

> If a situation is described by an equation in the form $y = kx$ where $k$ is a nonzero constant, we say that ***y varies directly as x*** or ***y is directly proportional to x***. The number $k$ is called the ***constant of variation*** or the constant of proportionality.

- Inverse variation.

> If a situation is described by an equation in the form $y = \frac{k}{x}$ where $k$ is a nonzero constant, we say that ***y varies inversely as x*** or ***y is inversely proportional to x***. The number $k$ is called the ***constant of variation.***

**Problem #9.** (Example #2, p. 355 , PART I).

Height, *H*, varies directly as foot length, *F*.

a) Write an equation that expressed this relationship.

b) Photographs of large footprints were published in 1951. Some speculated that these footprints were made by the Abominable Snowman.
Each footprint was 23 inches long.
The Snowman's height was determined to be 154.1 inches.
Use the $H = 154.1 \text{ and } F = 23$ to find the constant of variation.

**Problem #10.**

a) *y* varies directly as *x*, $y = 55$ when $x = 2.5$.
Find *y* when $x = 12$.

b) *y* varies inversely as *x*, $y = 55$ when $x = 2.5$.
Find *y* when $x = 12$.