Math 110
CH.2.4, 2.5. More INFO about graphs of functions.
Basic Elementary Functions.
Transformations of the Functions. **Lectures #9 and #10.**

- Obtaining Information from a Function's Graph.

- Summary about using closed dots, open dots, and arrows on the graphs.

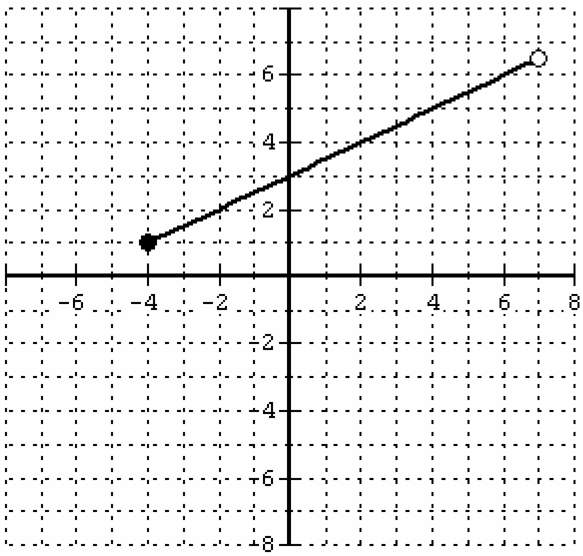
1. A closed dot indicate that the graph does not extend beyond this point and the point belongs to the graph.

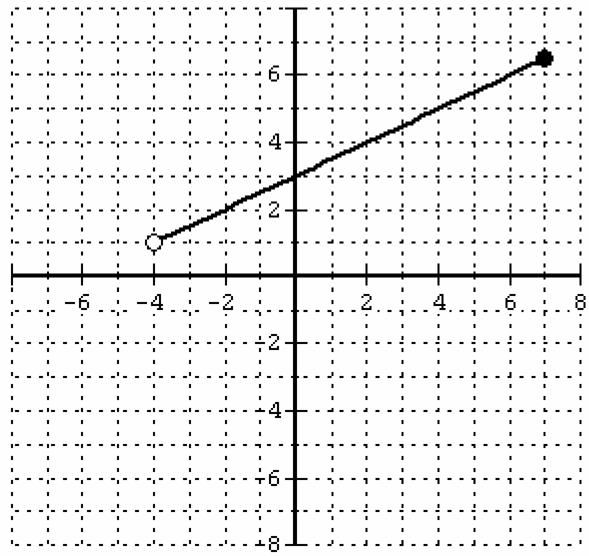
2. An open dot indicates that the graph does not extend beyond this point and the point does not belong to the graph.

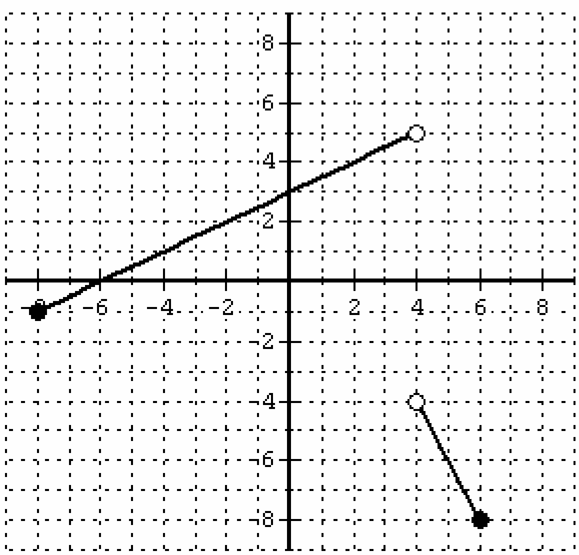
3. An arrow at the right or left of the graph indicates that the graph extends indefinitely in the direction in which the arrow points.

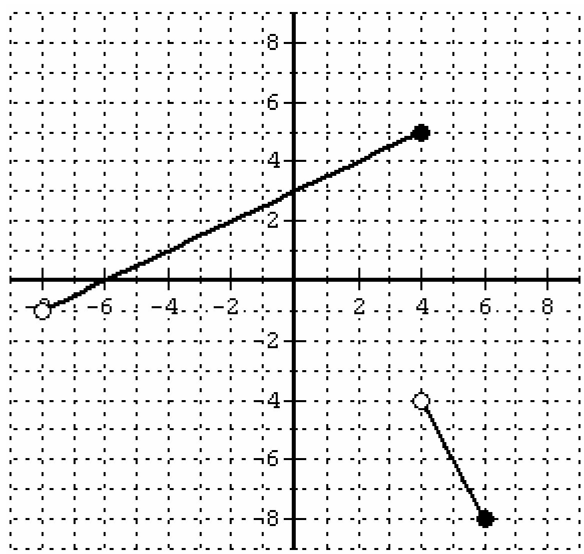
Problem #1. Find the domain and range for the functions whose graphs are shown below.

Math 110
CH.2.4, 2.5. More INFO about graphs of functions.
Basic Elementary Functions.
Transformations of the Functions. Lectures #9 and #10.









- Graph's Intercepts.

Definitions of *y*-intercept.
The *y*-coordinate of the point where a function's graph intersects the *y*-axis is called the *y*-intercept of the graph. Every function can have at most one *y*-intercept.

y-intercept = $f(0)$ according to the definition above.

Definitions of *x*-intercept(s).
The *x*-coordinates of the points where a function's graph intersects the *x*-axis is called the *x*-intercepts of the graph (**zeros of the function**).
A function can have more than one *x*-intercepts.

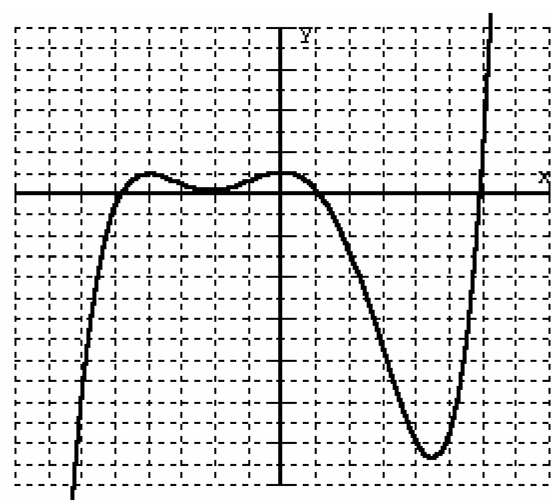
To find *x*-intercepts, we have to set up the equation $f(x)=0$ and solve it.

Problem #2. Find the intercepts for following functions.

a) $y = -5x + 3$

b) $f(x) = x^2 - 6x + 9$

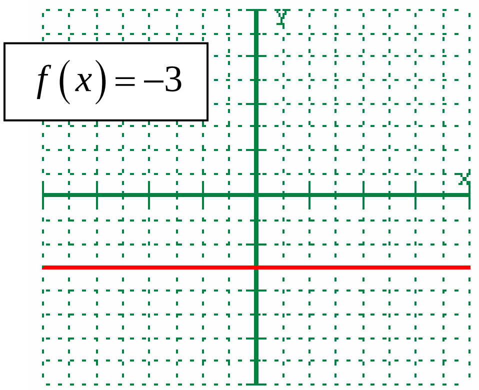
c)



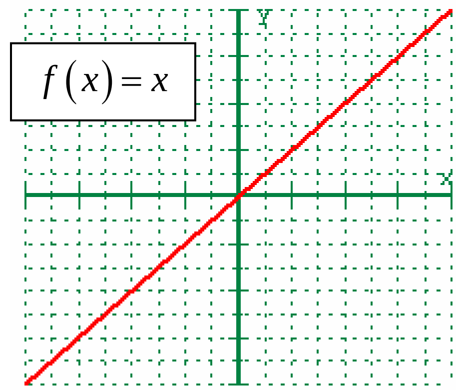
Math 110
CH.2.4, 2.5. More INFO about graphs of functions.
Basic Elementary Functions.
Transformations of the Functions. **Lectures #9 and #10.**

▪ Basic elementary functions.

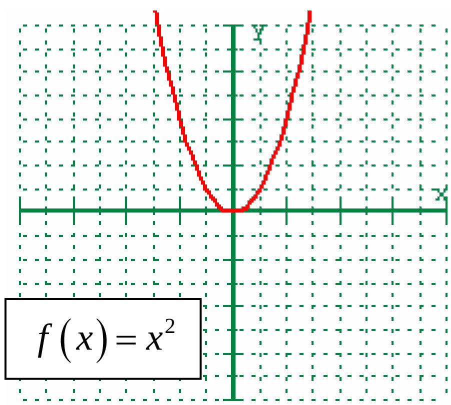
Constant function



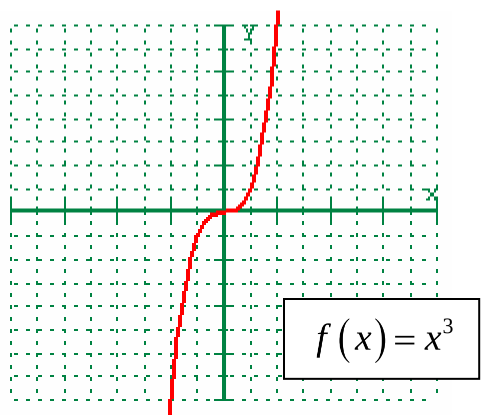
Identity function



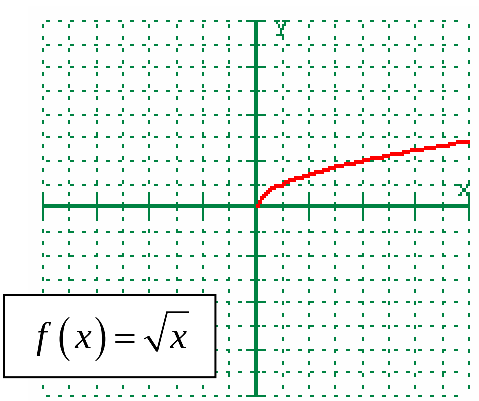
Basic Quadratic Function



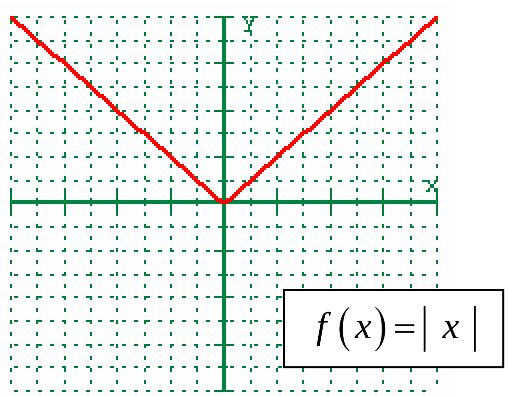
Basic Cubic function



Square Root Function



Absolute Value Function



- Even and Odd Functions and Symmetry.

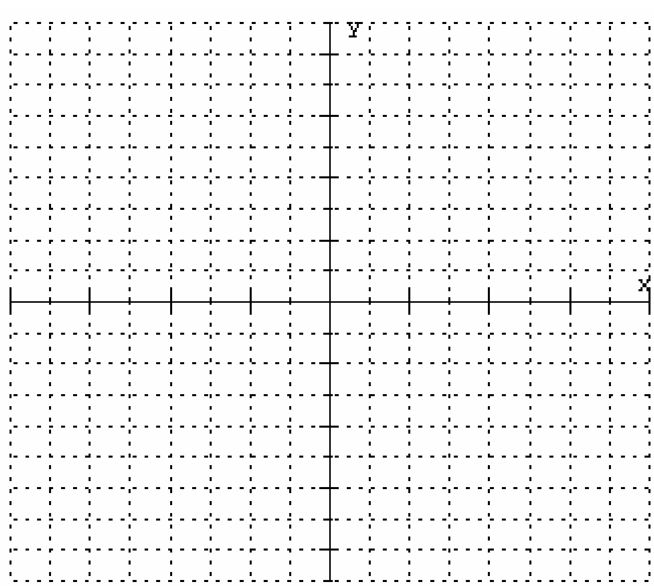
The function f is an even function if $f(-x) = f(x)$ for all x in the Domain of *f*.
The graph of every even function has a symmetry about *y*-axis.

The function f is an odd function if $f(-x) = -f(x)$ for all x in the Domain of *f*.
The graph of every odd function has a symmetry about origin.

<u>Problem#3.</u> Which of basic elementary functions are even? odd? neither even nor odd?

- Functions transformations and graphs.

Sketch the graphs of the functions $y = x^2$, $y = x^2 + 3$, $y = x^2 - 3$.



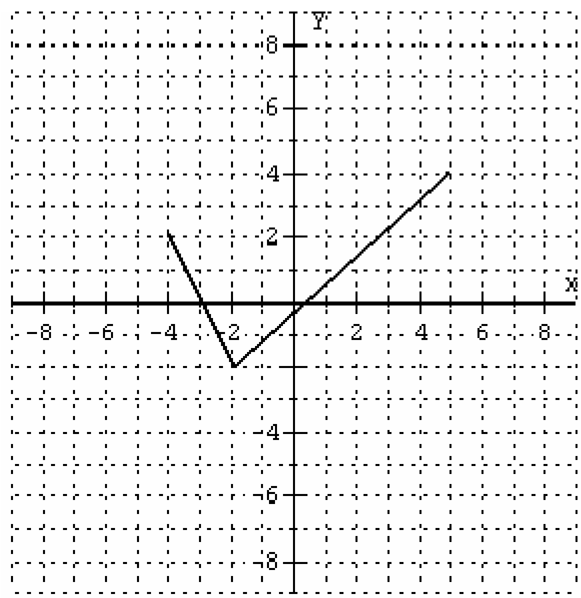
- **Vertical Translations**.

If f is a function and c is a positive constant, then the graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted up vertically c units, and the graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted down vertically c units.

In coordinates: If the point (x_0, y_0) in on the graph of f, then the point $(x_0, y_0 \pm c)$ is on the graph of $y = f(x) \pm c$.

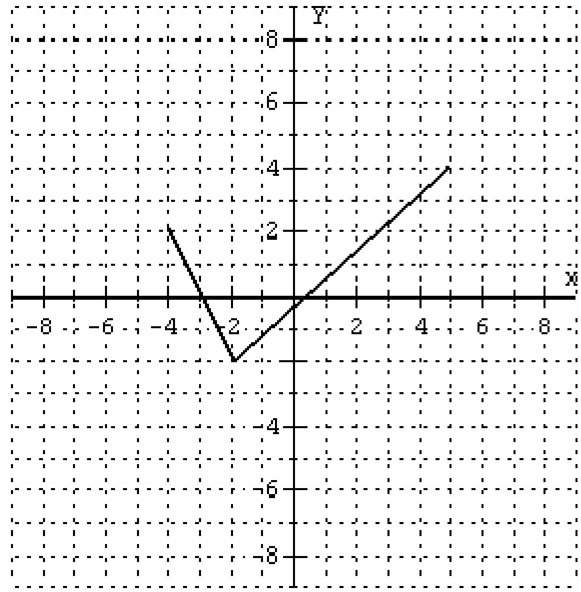
Problem #4.

Given the graph of a function $y = f(x)$.
Sketch the graph of the function $y = f(x) - 2$.
State the domain and range for each function.

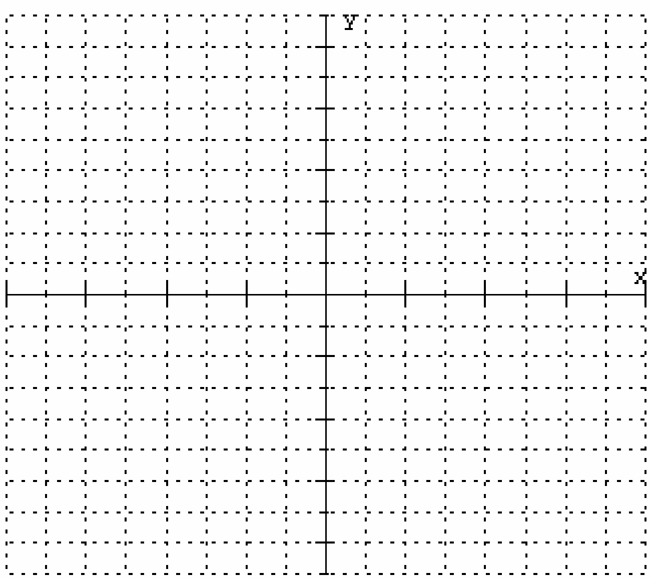


Problem #5.

Given the graph of a function $y = f(x)$.
Sketch the graph of the function $y = f(x) + 2$.
State the domain and range for each function.



- Sketch the graphs of the functions $y = x^2$, $y = (x-2)^2$, $y = (x+2)^2$.



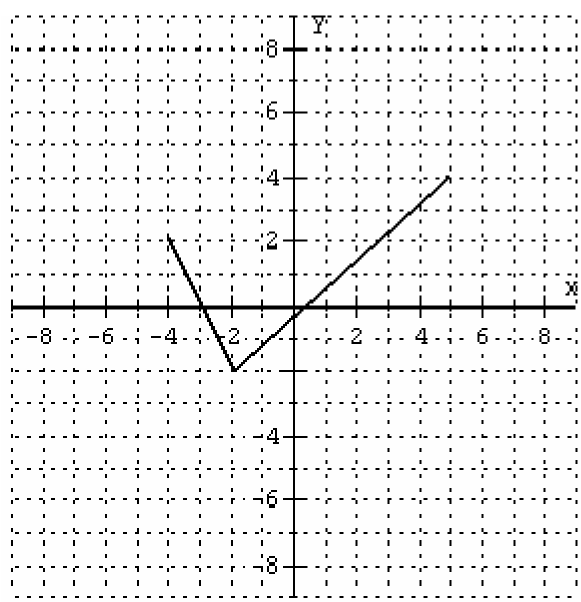
- **Horizontal Translations.**

If f is a function and c is a positive constant, then the graph of $y = f(x+c)$ is the graph of $y = f(x)$ shifted left horizontally c units, and the graph of $y = f(x-c)$ is the graph of $y = f(x)$ shifted right horizontally c units.

In coordinates: if the point (x_0, y_0) in on the graph of f, then the point $(x_0 \mp c, y_0)$ is on the graph of $y = f(x \pm c)$.

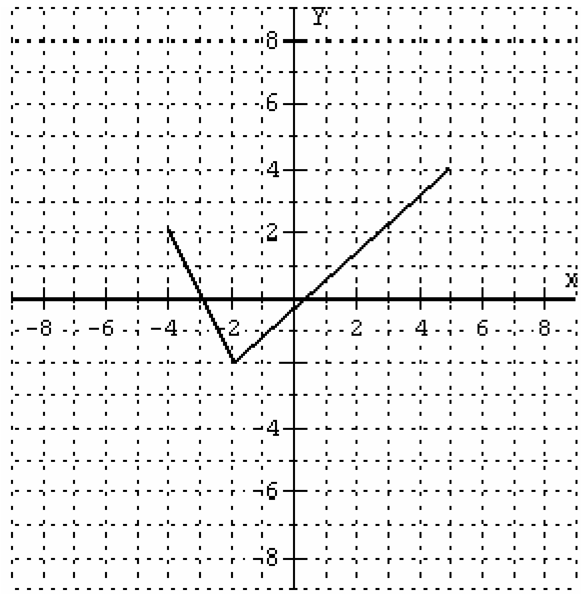
Problem #6.

Given the graph of a function $y = f(x)$.
Sketch the graph of the function $y = f(x+1)$.
State the domain and range for each function.



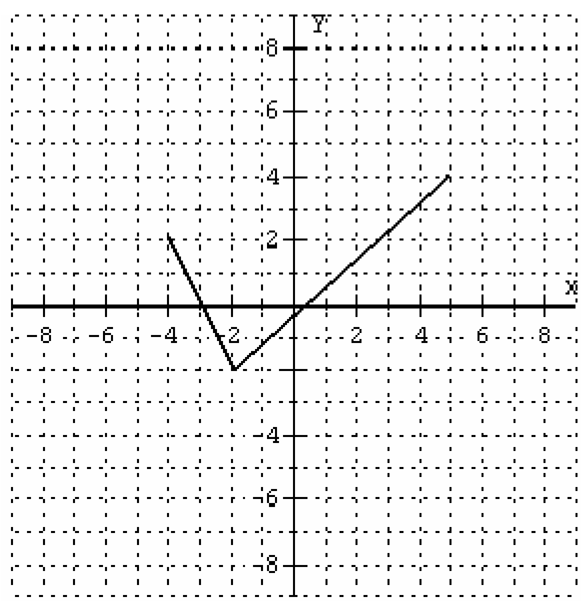
Problem #7.

Given the graph of a function
$y = f(x)$.
Sketch the graph of the function
$y = f(x-1)$.
State the domain and range
for each function.



Problem #8.

Given the graph of a function
$y = f(x)$.
Sketch the graph of the function
$y = f(x+1)-2$.
State the domain and range
for each function.



❖ Reflections of Graphs.

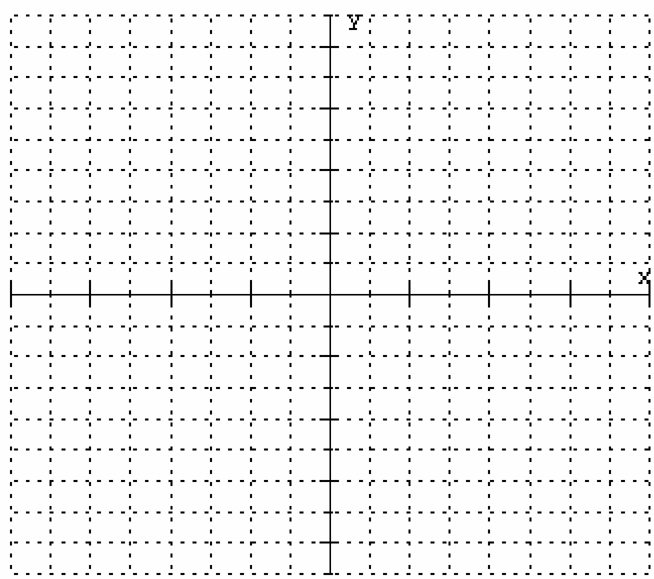
The graph of

- $y = -f(x)$ is the graph of $y = f(x)$ reflected across the *x*-axis.
- $y = f(-x)$ is the graph of $y = f(x)$ reflected across the *y*-axis.

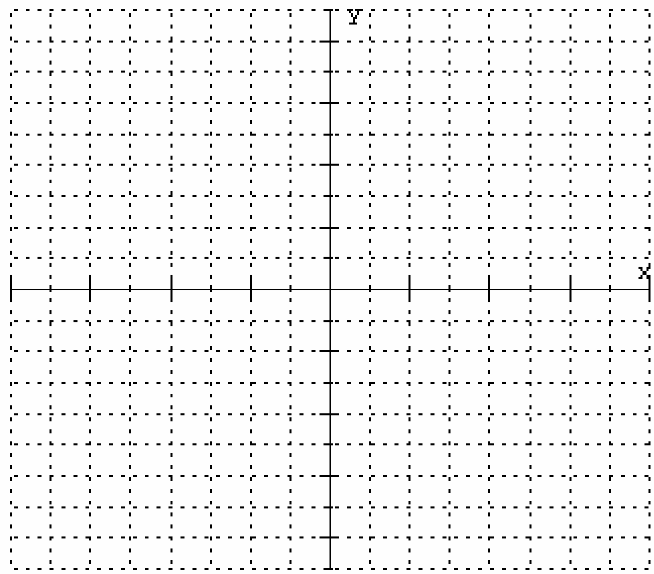
In coordinates: if the point (x_0, y_0) in on the graph of f then the point $(x_0, -y_0)$ is on the graph of $y = -f(x)$, and $(-x_0, y_0)$ is on the graph of $y = f(-x)$.

Problem #9.

Sketch the graphs of $y = x^2$ and $y = -x^2$.
State the domain and range for each function.
(Each square is one by one unit.)



Problem #10.

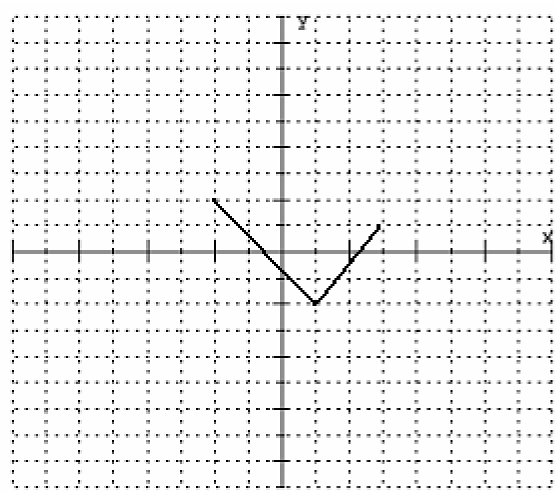


Sketch the graphs of $y=\sqrt{x}$, $y=\sqrt{-x}$, and $y=-\sqrt{x}$.

State the domain and range for each function

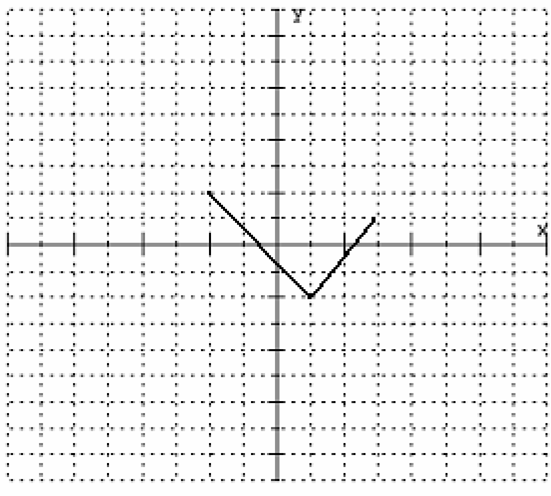
Problem #11.

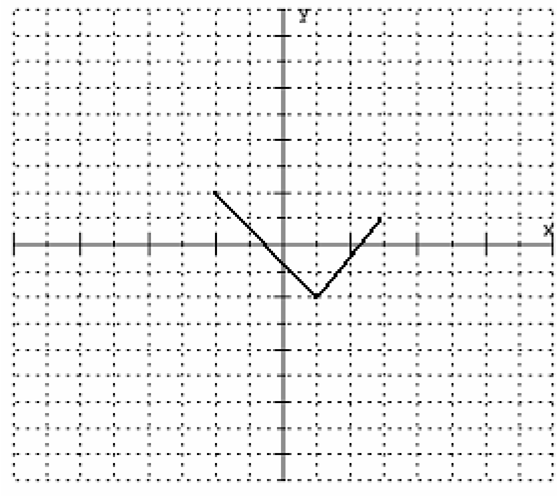
Given the graph of a function f, sketch the graph of



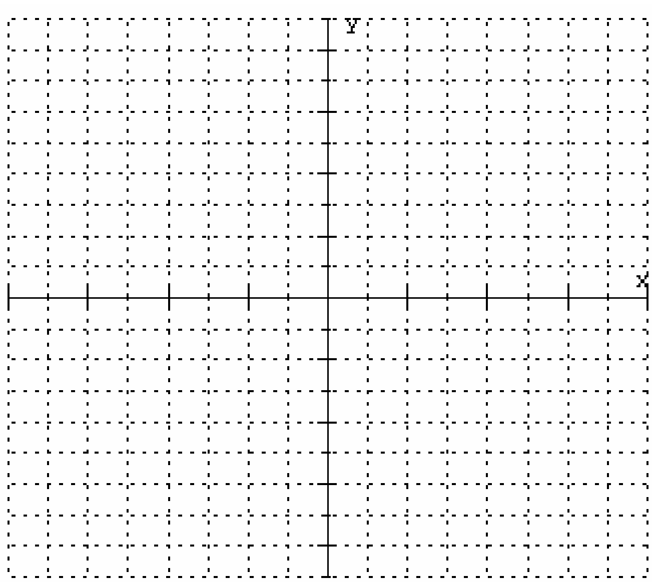
a) $y=f(-x)$,
b) $y=-f(x)$,

c) State the domain and range for each function.
Each square is one by one unit.





- Nonrigid transformations.
  Stretching and Shrinking (vertically) graphs.
  - Sketch the graphs of the functions $y = x^2$, $y = \frac{1}{2}x^2$, $y = 2x^2$.



- Given a function $y = f(x)$.

  Will consider the new function $y = c\, f(x)$, where c is a positive constant, $c \neq 1$.

  If $c < 1$, the graph of the new function $y = c\, f(x)$ is a **vertically compressed (shrunk)** version of the graph of $y = f(x)$.

  If $c > 1$, the graph of the new function $y = c\, f(x)$ is a **vertically stretched** version of the graph of $y = f(x)$.

Stretching and shrinking (compression) change the shape of the graph, thus these transformations are called **nonrigid**.

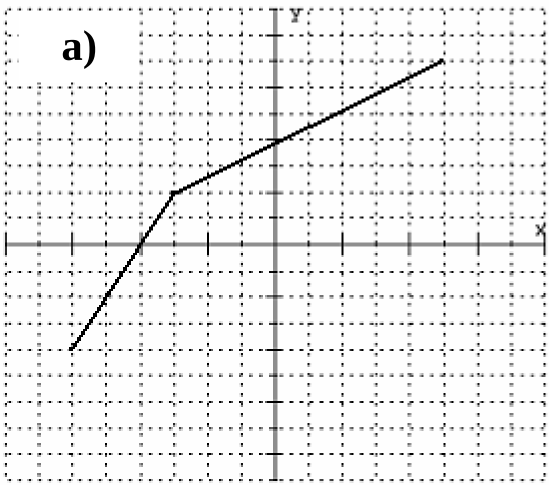
<u>In coordinates :</u> if the point (x_0, y_0) in on the graph of f, then the point $(x_0, c\, y_0)$ is on the graph of $y = c\, f(x)$.

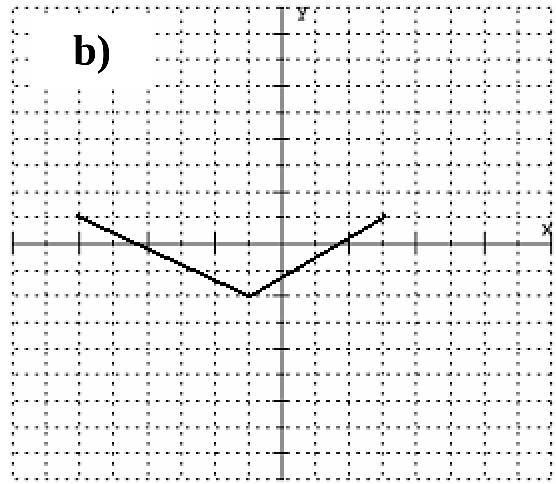
<u>Problem #12.</u>

Given the graph of a function f, sketch the graph of

a) $y = \dfrac{1}{2} f(x)$, b) $y = 3f(x)$.

c) State the domain and range for the original and transformed functions. (Each square is one by one unit.)





❖ <u>Sequence of Transformations.</u>

A function involving more than one transformations of some basic function can be graphed in the following order:

1. Horizontal translation (H).
2. Stretching or shrinking (S).
3. Reflection (R).
4. Vertical translation (V).

Remember this sequence: **H- S –R –V**

Problem #13.

Write the sequence of transformations which leads from
$f(x)=\sqrt{x}$ to $g(x)=3-\frac{1}{2}\sqrt{x+2}$.

Problem #14.

Write the sequence of transformations which leads from
f to $g(x)=-\frac{1}{2}f(x-3)+5$.

Problem #15.

Given the graph of a function f, sketch the graph of
$g(x)=-f(x)+2$