Definition of a Function.

A *function* is a rule (process or method) that produces a correspondence between two sets of elements such that to each element in the first set there corresponds one and only one element in the second set. The first set is called the *domain*, and the second set is called the *range*.

Examples. Functions may be defined by formula, by graph, by table, by description.

Functions defined by equations. We will consider only the case when D and R are sets of real numbers.

Definition. If in an equation in two variables, for each value of one variable we get exactly one corresponding value of another variable, then the equation defines a function.

<u>Terminology</u>. Independent variable – dependant variable

Input – output

Argument – function

Function notation. y = f(x). f is a symbol for the rule;

f(x) represents the element in the Range of f corresponding to x in the Domain of f. f(x) is **the**

value of the function f at x.

Domain. When a function is defined by a formula, the Domain usually means so-called "implied" or "natural " domain, which is the largest set of real numbers (x values) such as corresponding outputs (y values) are also real numbers.

<u>Range.</u> Set of all f(x) (y values) corresponding to all x from the Domain.

Problem #1.

Find the domain for the following functions.

a)
$$f(x) = 3x+9$$
, b) $g(x) = x^2 + 5x - 11$
c) $f(x) = \frac{5x}{x^2 - 25}$ d) $y = \sqrt{3x-2}$
e) $y = \frac{x-4}{\sqrt{x+2}}$.

Problem #2

Find the domain of $y = 3x^5 - 2x^4 - 5x^3 + x^2 - 7x + 100$.

Problem #3.

y = 553.37x + 27,966 gives the annual teacher's salary in the USA *x* years after 1985. Estimate teacher's salary in 2002, 2010.

• Graph of a Function.

The graph of the function f is the set of points (x, f(x)) such that x is in the domain of f.

Graphing Functions by Plotting Ordered Pairs.

Problem #4.

x	-3	-2	-1	0	1	2	3
f(x)							

Graph each f(x) f(x

a)
$$f(x) = x^2$$

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b)
$$f(x) = 2x - 1$$

 x

 $f(x)$

 $f(x)$

 x

 $f(x)$

 x

 $f(x)$

 $f(x)$

 x

 $f(x)$

 f









Vertical line test.

An equation defines a function if each vertical line in the coordinate system passes through at most one point on the graph of the equation.

Piecewise functions.

Definition. Functions whose definitions involve more than one rule are called Piecewise-defined Functions.

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<u>Examples.</u>



d)Question:Is the functiondefined at *x* = 1?



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Problem #5.

Given
$$f(x) = \begin{cases} 2x - 3 & x \le -1 \\ x^2 - 4 & x > -1 \end{cases}$$

a) Find the domain D and range R; b) f(0); c) f(2) - f(-2).

Problem #6.

The graph of a piecewise defined function is shown on the right. Find according to this graph a) f(0) =

b)
$$f(1) =$$

c) f(2) - f(-1) =

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• Difference quotient
$$\frac{\Delta f}{h}$$
, $h \neq 0$

Definition. $\frac{f(x+h) - f(x)}{h}, h \neq 0, \text{called difference}$ quotient or average rate of change for the function y = f(x).

Problem #7.

Given
$$f(x) = 5x - 3$$
.

Find the following and simplify when it is possible.

a)
$$f(2) =$$

b)
$$f(2+h) =$$

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c)
$$f(2+h) - f(2) =$$

d) $\frac{f(2+h) - f(2)}{h} =$
e) $f(a) =$
f) $f(-a) =$
g) $f(a+h) - f(a) =$
h) $\frac{f(a+h) - f(a)}{h} =$

h

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Problem #8. Given $f(x) = 2x^2 - 3$.

Find the following and simplify when it is possible.

a) f(2) =

c)
$$f(2+h) =$$

c)
$$f(2+h) - f(2) =$$

d)
$$\frac{f(2+h)-f(2)}{h} =$$

e)
$$f(a) =$$

f)
$$f(-a) =$$

g)
$$f(a+h)-f(a) =$$

h)
$$\frac{f(a+h)-f(a)}{h} =$$

Problem #9. Given $f(x) = \frac{1}{x-3}$.

Find the following and simplify when it is possible.

a)
$$f(5) =$$

b) $f(5+h) =$
c) $f(5+h) - f(5) =$
d) $\frac{f(5+h) - f(5)}{h} =$
e) $f(a) =$
f) $f(-a) =$
g) $f(a+h) - f(a) =$

h)
$$\frac{f(a+h)-f(a)}{h} =$$