- Introduction to Quadratic Equations.

Definition of a quadratic equation.
A quadratic equation in *x* is an equation that can be written in the form
$ax^2 + bx + c = 0, \quad where\ \ a, b, \ \ and\ \ c\ are$
$real\ \ numbers\ \ with\ \ a \neq 0.$

A quadratic equation in *x* also called a *second-degree polynomial equation* in *x*.

Problem #1. Which of the following are Quadratic Equations in *x*?

$a)\ \ x^2 + 3x^4 - 2x = 7;$ $\qquad b)\ \ x^2 + 3x = 7;$

$c)\ \ x^2 + 3x = 0;$ $\qquad d)\ x^2 = 7;$

$e)\ \ \dfrac{x^2}{3} - x + 2 = 0;$ $\qquad f)\ \ \dfrac{3}{x^2} - 2x + 4 = 0$

- Different methods for solving Quadratic Equations.

1. Factoring.
Factored Quadratic Equation can be solved using the **Zero Product Principle.**

> If the product of two numbers (variables, algebraic expressions)
> $A \cdot B = 0$, then
> $A = 0 \;\; or \;\; B = 0 \;\; or \; A \;\; and \;\; B \; are \;\; both \;\; 0.$

<u>**Problem #2.**</u> Solve the following equations by factoring, using the Zero Product Principle.

a) $3x^2 + 13x - 10 = 0;$ b) $5x^2 - 3x = 0;$

c) $x^2 - 3 = 0;$ d) $2x\left(x+5\right) - 5x = 3$

e) $5x^2 - 3x - 2 = 0;$ f) $x^2 + 3 = 0$

Strategy for solving QE by factoring.

1. Move all terms in one side (thus another side is 0).
2. Factor.
3. Apply the Zero Product Principle, setting each factor (linear) $= 0$.
4. Solve two linear equations.
5. Check (by substitution into the original quadratic equation) is optional.

Note: Always check your factoring by distribution.

Chapter P.5 is about Factoring. HW for P.5 helps to build technique.
Use this CH. for reviewing the material and exercises.

Question. Is it possible to solve *any* Quadratic Equation by factoring?

2. Square Root Method.

If u is an algebraic expression and d is a positive real number, then
the equation $u^2 = d$ has exactly two solutions:

$$u = \sqrt{d}, \quad and \quad u = -\sqrt{d} \quad \left(u = \pm\sqrt{d}\right)$$

<u>Problem #3.</u> Solve the following equations:

a) $x^2 = 25$; b) $9x^2 = 5$**;** c**)** $(4x-3)^2 = 16$

The equation $x^2 = k$, where $k < 0$ has no real solutions.

3. Completing the square procedure.

Change the quadratic equation in the form
$ax^2 + bx + c = 0$
to an equivalent equation in the form
$a(x-d)^2 = k$ which then can be solved using the Square Root Method.

<u>Problem #5.</u> Change given Quadratic Equation to the equivalent form with complete square and solve it.
$9x^2 - 6x - 4 = 0$

4. Quadratic formula.

When "Completing the Square" procedure is applied to a quadratic equation in general form, $ax^2 + bx + c = 0$, then we receive the Quadratic formula - the expression for the solutions of a Quadratic Equation through coefficients of this equation.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Problem #6.</u> Solve the following equations using the Quadratic Formula.

a) $3x^2 + 2x - 5 = 0$; b) $2x^2 + 5x - 3 = 0$;

c) $x^2 + x - 1 = 0$; d) $x^2 - 6x + 9 = 0$

Analysis of the results for a) - d).

Solution set for a Quadratic Equation may contain
1) two distinct real numbers (cases a) and b));
2) one real number (repeated root);
3) no real solution.

- **The Discriminant and the Nature of Roots.**

The Discriminant of $ax^2 + bx + c = 0$ is $D = b^2 - 4ac$.

- Connection between the value of *D* and nature of roots of Quadratic Equations.

$D > 0$ two distinct real zeros
$D = 0$ one (repeated) real zero
$D < 0$ no real solution

Problem #8. Without solving the equation, determine the number of its roots and the nature of these roots.

a) $x^2 + x + 1 = 0$ b) $3x^2 + 5x - 9 = 0$

c) $x^2 - 3x + 2 = 0$ d) $x^2 - 6x + 9 = 0$