Factoring Polynomials.

- Definition of a Polynomial in *x*.

> A **polynomial in *x*** is an algebraic expression in the form
> $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0,$
> where
> $a_n,\ a_{n-1},\ a_{n-2},\ \ldots\ ,\ a_1,$ and a_0 are real numbers, $a_n \neq 0,$
> n is a nonnegative integer.
> n is the degree of polynomial
> a_n is the leading coefficient,
> a_0 is the constant term.

- Factoring is the process of writing a polynomial as the product of two or more polynomials. We will do factoring with ***integer coefficients***. Polynomials that cannot be factored using integer coefficients are called ***irreducible over the integers, or prime.***

- Methods of Factoring.

A. Factoring out the Greatest Common Factor.

<u>Problem #1.</u> Factor

a) $25x^5 - 15x^3$ b) $3x(x-2) - 24(x-2)$

B. Factoring by grouping.

Problem #2. Factor

$x^4 - 5x^3 - 3x + 15$

C. Factoring Trinomials $ax^2 + bx + c$.

Problem #3. Factor

a) $x^2 + 5x + 6$ b) $6x^2 + 13x - 5$

D. Factoring the Difference of Two Squares.

$$A^2 - B^2 = (A+B)(A-B)$$

Problem #4. Factor

a) $121x^2 - 4y^2$ b) $x^2 - 5$

E. Factoring Perfect Square Trinomials.

$$A^2 + 2AB + B^2 = (A+B)^2$$
$$A^2 - 2AB + B^2 = (A-B)^2$$

Problem #5. Factor

a) $x^2 - 10x + 25$ b) $2x^3 + 12x + 18$

F. Factoring Sums and Differences of Two Cubes.

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

Rational Expressions.

- Definition.
  A Rational Expression is the quotient of two polynomials.
  Examples:

$$\frac{x^2 - 2x + 5}{x - 1}, \quad \frac{1}{x}, \quad \frac{x-1}{5}, \quad \frac{x+4}{x^3 + x^2 + x + 1}$$

- The Domain of the Rational Expression.

The **Domain** of the Rational Expression is the set of real numbers for which the expression is defined.

Problem #6. Find the domain for the following rational expressions.

a) $\dfrac{x^2-2x+5}{x-1}$ b) $\dfrac{1}{x}$ c) $\dfrac{x}{x^2+5x+6}$

- Simplifying Rational Expressions.

A Rational Expression is simplified if its numerator and denominator have no common factors other than 1 or -1.

- Simplifying Rational Expressions.
  1. Factor the numerator and denominator completely.
  2. Divide both the numerator and denominator by the common factors.

Problem #7. Simplify the following rational expressions.

a) $\dfrac{x+2}{x^2-4}$ b) $\dfrac{x^2-2x+1}{x^2-1}$

- Arithmetic with Rational Expressions.

A. We can **add** and **subtract** rational expressions with the same denominator, thus we need to find the Least Common Denominator and re-write in terms of Least Common Denominator.

Problem #8. Perform operations.

a) $\frac{2}{x+3}-\frac{x}{x+3}$ b) $\frac{2}{x+3}+\frac{x}{x-3}$ c) $\frac{x}{x^2-2x}-\frac{1}{x+1}$

B. Multiplication.

The product of two rational expressions is the product of their numerators divided by the product of their denominators.

Problem #9. Perform multiplication. Simplify your answer.

$$\frac{x-5}{x+2}\cdot\frac{x+6}{x^2-25}$$

C. Dividing rational expressions.

The quotient of two rational expressions is the product of first expression and reciprocal of second.

Problem #10. Divide and simplify

$$\frac{x-3}{x^2-1}\div\frac{x^2-9}{x^2-2x+1}$$

- Simplifying Complex Rational Expressions (complex fractions).

  Main idea: Re-write the numerator and denominator of a given rational expression as a single term, then perform the division.

Problem #11. Simplify.

$$\frac{\frac{x}{x-2}+1}{\frac{x}{x^2-4}+1}$$