Math 110 SS 2007
Real numbers. **HO #1 (for lectures ##1-2).**

THE REAL NUMBER SYSTEM

The real number system evolved over time by expanding the notion of what we mean by the word "number." At first, "number" meant something you could count, like how many sheep a farmer owns. These are called the *natural numbers*, or sometimes the *counting numbers*.

Natural Numbers

or "Counting Numbers"

1, 2, 3, 4, 5, . . .

At some point, the idea of "zero" came to be considered as a number. If the farmer does not have any sheep, then the number of sheep that the farmer owns is zero. We call the set of natural numbers plus the number zero the *whole numbers*.

Whole Numbers

Natural Numbers together with "zero"

0, 1, 2, 3, 4, 5, . . .

Integers

Whole numbers plus negatives

. . . –4, –3, –2, –1, 0, 1, 2, 3, 4, . . .

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Rational Numbers

All numbers of the form $\frac{a}{b}$, where *a* and *b* are integers (but *b* cannot be zero).

Rational numbers include what we usually call *fractions*

- *Notice that the word "rational" contains the word "ratio," which should remind you of fractions.*

The bottom of the fraction is called the *denominator*. Think of it as the *denomination*—it tells you what size fraction we are talking about: fourths, fifths, etc.
The top of the fraction is called the *numerator*. It tells you *how many* fourths, fifths, or whatever.

Irrational Numbers

- Cannot be expressed as a ratio of integers.
- As decimals they never repeat or terminate (rationals always do one or the other)

Examples:

$\frac{3}{4} = 0.75$	Rational (terminates)
$\frac{2}{3} = 0.66666\overline{6}$	Rational (repeats)
$\frac{5}{11} = 0.45454\overline{45}$	Rational (repeats)
$\sqrt{2} = 1.41421356...$	Irrational (never repeats or terminates)
$\pi = 3.14159265...$	Irrational (never repeats or terminates)

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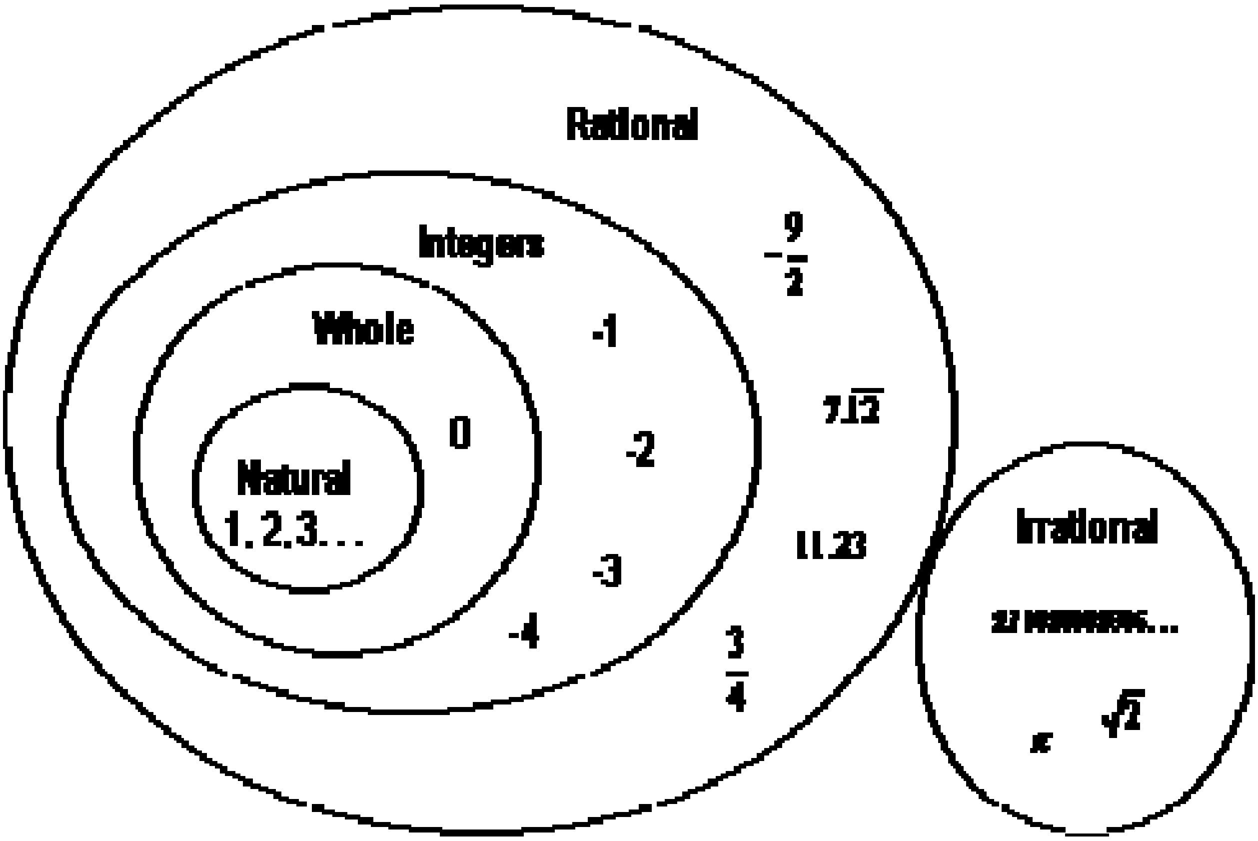
The Real Numbers

- Rationals + Irrationals
- All points on the number line

When we put the irrational numbers together with the rational numbers, we finally have the complete set of real numbers. Any number that represents an amount of something, such as a weight, a volume, or the distance between two points, will always be a real number. The following diagram illustrates the relationships of the sets that make up the real numbers.



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An Ordered Set

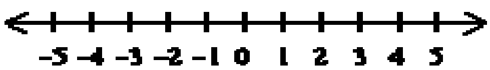
The real numbers have the property that they are *ordered*, which means that given any two different numbers we can always say that one is greater or less than the other. A more formal way of saying this is:

For any two real numbers a and b, one and only one of the following three statements is true:

1. a is less than b, (expressed as $a < b$)
2. a is equal to b, (expressed as $a = b$)
3. a is greater than b, (expressed as $a > b$)

The Number Line

The ordered nature of the real numbers lets us arrange them along a line (imagine that the line is made up of an infinite number of points all packed so closely together that they form a solid line). The points are ordered so that points to the right are greater than points to the left:



- Every real number corresponds to a distance on the number line, starting at the center (zero).
- Negative numbers represent distances to the left of zero, and positive numbers are distances to the right.
- The arrows on the end indicate that it keeps going forever in both directions.

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Absolute Value

The absolute value of a number is the distance from that number to the origin (zero) on the number line. That distance is always given as a non-negative number.

Formal definition.

$$|\ a\ | = \begin{cases} a, & a \geq 0 \\ -a, & a > 0 \end{cases}$$

Examples

$|15| =$

$|-\ 25| =$