Any of the following exercises are fair game for appearing in a slightly altered state on an exam. I suggest that you write up your solutions **neatly** in your own handwriting, or .tex them up, so that you can easily use them for studying later. All solutions should be your own! Feel free to discuss solutions with others, but don't write anything down that you don't understand (and write it all yourself!). **Only a subset of the problems will be graded, so do them all to the best of your ability, and be aware: even if there is only one problem that you don't do well, it might be one that's graded! So, try to do them all well!**

PROBLEMS RELATED TO FOLLAND CHAPTER 1 AND SECTIONS 2.1 & 2.2

1. Use trigonometric polynomials to solve the partial differential equation
$$\frac{\partial}{\partial t}u \ = \ 5\frac{\partial^2}{\partial x^2}u - \cos(3x),$$
where $u(x,0) = \sin(x)$.

2. Verify formula 7 in Table 1 on page 26 of Folland. Recall that the Folland *.pdf is available on D2L...

3. Verify formula 8 in Table 1 on page 26 of Folland.

4. Verify formula 19 in Table 1 on page 28 of Folland.

5. Use formula 8 in Table 1 of Folland to derive formula's 9 and 10 in Table 1 (indirectly).

6. Do Exercise 1 on page 37 of Folland.

7. Do Exercise 2 on page 37 of Folland.

8. Do Exercises 3, 4, and 5 on page 37 of Folland.

9. Do Exercises 6 and 7 on page 37 of Folland.

10. Consider the function $f(x) = \sin(1/x)$ on $[-\pi, \pi]$, where we define $f(0) = 0$.
    (a) Sketch this rather strange function. Is it Riemann integrable? Why, or why not?
    (b) Prove that it's Fourier coefficients $c_n$ approach zero as $n \to \infty$.

11. Now consider the function $f(x) = 1/|x|^{\frac{1}{4}}$ on $[-\pi, \pi]$, where we again define $f(0) = 0$. Bessel's Inequality on page 30 of Folland does not directly apply to this function (since it is unbounded it is only integrable as an improper integral, i.e., as the limit of integrals of Riemann integrable functions, and so is not Riemann integrable in its own right). We will have a general theory for functions like this later in the course. For now, we can prove a few things about this specific $f$.
    (a) Prove that every Fourier coefficient, $c_n$, of $f$ exists.
    (b) Do the Fourier coefficients exist for its derivative, $df/dx$, (again with $df/dx$ set to 0 at 0)?
    (c) Suppose that $f(x) = \sum_{n=-\infty}^{\infty} c_n \mathbb{e}^{\mathbb{i}nx}$ for all $x \in (0, 1)$. Does $\sum_{n=-\infty}^{\infty} |c_n|$ converge?
    (d) Based on what we know so far, can we claim that the Fourier coefficients of $f$, $c_n$, do not approach zero as $n \to \infty$. Why, or why not?