## Exercises: (Chapter 7 §2 and §3)

1. Implement an algorithm in the programming language of your choice that computes

$$\int_0^x e^{-t^2}\,dt$$

by summing an approximate Taylor series until the individual terms fall below $10^{-8}$ in magnitude. Test your program by using it to approximate this integral for $x = 0.0, 0.1, 0.2, \ldots, 1.0$.
**Turn in both your code and the integral values you computed.**

2. Prove (without using it's error term) that Simpson's Rule,

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right],$$

correctly integrates all cubic polynomials $f(x) = ax^3 + bx^2 + cx + d$.

3. Find a formula of the form

$$\int_0^{2\pi} f(x)\,dx \approx A_1 f(0) + A_2 f(\pi)$$

that is exact for any function having the form $f(x) = a + b\cos(x)$. Then, prove that the resulting formula is also exact for any function of the form

$$f(x) = \sum_{k=0}^{n}\left[a_k \cos\left((2k+1)x\right) + b_k \sin\left(kx\right)\right].$$

4. Find values for $c$, $x_0$, and $x_1$ so that the formula

$$\int_0^1 f(x)\,dx \approx c\left[f(x_0) + f(x_1)\right]$$

is exact for all $f \in \Pi_3$.

5. Find a formula of the form

$$\int_0^1 x f(x)\,dx \approx A_0 f(x_0) + A_1 f(x_1)$$

that is exact for all polynomials of degree 3 (i.e., find values for $A_0$, $A_1$, $x_0$, and $x_1$).