x	‖	0	1	2
$p(x)$	‖	2	-4	44
$p'(x)$	‖	-9	4	

Table 1: The data for Problems 2 and 3.

Exercises: (Chapter 6 §2 – §4)

1. Use the computer language/platform of your choice to compute the degree n polynomial p_n that interpolates $f(x) = 1/(1+x^2)$ evaluated on the interval $[-5, 5]$ at the $n+1$ equally spaced nodes $x_j = -5 + 10j/n$ for $j = 0, \dots n$. Then, use your code to answer the following questions.
   (a) Compute $p_5(x)$. What is its maximum error from f on 31 equally spaced grid points in $[-5, 5]$, $\max\{|f(y_\ell) - p_5(y_\ell)| \text{ s.t. } y_\ell = -5 + \ell/3,\ \ell = 0, \dots, 30\}$?
   (b) Compute $p_{10}(x)$. What is its maximum error from f on 31 equally spaced grid points in $[-5, 5]$, $\max\{|f(y_\ell) - p_{10}(y_\ell)| \text{ s.t. } y_\ell = -5 + \ell/3,\ \ell = 0, \dots, 30\}$?
   (c) Compute $p_{15}(x)$. What is its maximum error from f on 31 equally spaced grid points in $[-5, 5]$, $\max\{|f(y_\ell) - p_{15}(y_\ell)| \text{ s.t. } y_\ell = -5 + \ell/3,\ \ell = 0, \dots, 30\}$?

   Make sure to turn in your code in addition to answering the three questions above!

2. Use the extended Newton divided difference method to obtain a quartic (i.e., degree 4) polynomial p that matches the data in Table 1.

3. Find a quintic (i.e., degree 5) polynomial p that matches the data in Table 1 and, in addition, satisfies $p(3) = 2$. *HINT: Add a suitable term to the polynomial found in Problem 2.*

4. Obtain a formula for the polynomial p of least degree that takes on these values:

$$p(x_j) = y_j \quad \text{and} \quad p'(x_j) = 0 \quad \text{for } 0 \le j \le n.$$

5. Using the development of cubic splines as a model, derive the appropriate equations and algorithm to provide a quadratic spline interpolant to data (t_j, y_j) for $0 \le j \le n$, where $t_0 < t_1 < \cdots < t_n$. If f is the spline interpolant of the data, then the numbers $z_j = f'(t_j)$ are well defined. **Find the equations governing** $z_0, z_1, \dots, z_n$. You should discover that one of these z_j points can be arbitrary, so that you can always let $z_0 = 0$.