x	‖	-2	0	1
f(x)	‖	0	1	-1

Table 1: The data for Problem 4.

x	‖	0	1	2	7
y	‖	51	3	1	201

Table 2: The data for Problem 6.

Exercises: (Chapter 3 §4 and Chapter 6 §1 and §2)

1. Let $p > 1$. Consider the continued fraction

$$x = \cfrac{1}{p + \cfrac{1}{p + \cfrac{1}{p + \cfrac{1}{p + \cdots}}}}.$$

   (a) Solve for what x must equal in terms of p using fixed point methods.

   (b) Use the Contractive Mapping Theorem to prove that the sequence of values converges.

2. Show that the following functions are contractive on the given domains, yet they have no fixed point on these domains. Why do they not contradict the Contractive Mapping Theorem?

   (a) $F(x) = 3 - x^2$ on $[-\frac{1}{4}, \frac{1}{4}]$.

   (b) $F(x) = -x/2$ on $[-2, -1] \cup [1, 2]$.

3. The polynomial p of degree $\leq n$ that interpolates a given function $f : \mathbb{R} \to \mathbb{R}$ at $n+1$ prescribed nodes is uniquely defined. Hence, there is a mapping $f \mapsto p$. Denote this mapping by L and show that

$$Lf = \sum_{j=0}^{n} f(x_j)\ell_j,$$

   in Lagrange form. Next, show that L is **linear**, meaning that $L(af + bg) = aLf + bLg$ holds for all $f, g : \mathbb{R} \to \mathbb{R}$ and $a, b \in \mathbb{R}$.

4. Find the Lagrange and Newton forms of the interpolating polynomial for the data in Table 1. Write both polynomials in the form $a + bx + cx^2$ to verify that they are identical as functions.

5. Prove that if p is a polynomial of degree at most n, then

$$p(x) = \sum_{j=0}^{n} p[x_0, x_1, \ldots, x_j] \sum_{k=0}^{j-1} (x - x_k).$$

6. Determine the Newton interpolating polynomial for the data in Table 2.

7. The polynomial $p(x) = 2 - (x + 1) + x(x + 1) - 2x(x + 1)(x - 1)$ interpolates the first four points in Table 3. By adding one additional term to p, find a polynomial that interpolates the whole table.

x	‖	-1	0	1	2	3
y	‖	2	1	2	-7	10

Table 3: The data for Problem 7.