Exercises: (Chapter 3 §1, §2, and §3)

1. Consider the bisection method starting with the interval $[1.5, 3.5]$.
   (a) What is the width of the interval at the n^{th} step of this method?
   (b) What is the maximum distance possible between the root r and the midpoint of this interval?

2. Code up the bisection method in the language of your choice and use it to answer the following two questions. Use "double" variables for all non-integers.
   (a) Find a root of the polynomial
   $$x^8 - 36x^7 + 546x^6 - 4536x^5 + 22449x^4 - 67284x^3 + 118124x^2 - 109584x + 40320$$
   in the interval $[5.5, 6.5]$ to within an error $< 10^{-8}$.
   (b) Change the -36 to -36.001 and repeat.

   Turn in your code along with a discussion of your results. How much did the root move?

3. Which of the following sequences converge quadratically as $n \to \infty$?
   (a) $\frac{1}{n^2}$
   (b) $\frac{1}{2^{2^n}}$
   (c) $\frac{1}{2^{n!}}$
   (d) $\frac{1}{\exp(n)}$
   (e) $\frac{1}{n^n}$

4. Consider the iteration formula
   $$x_{n+1} = x_n - \frac{f(x_n)}{g(x_n)}$$
   where
   $$g(x) = \frac{f\left(x + f(x)\right) - f(x)}{f(x)}.$$
   Assume that $f \in C^2([a, b])$ and that (a, b) contains a simple zero, r, of f. Show that the iteration converges to r quadratically provided that x_0 is initialized close enough to r.

5. Write down a Newton iteration for computing the fifth root of any positive real number c. Importantly, make it clear what function you're finding a zero of with your iteration (i.e., show your work and explain your thinking!).

6. If the secant method is applied to the function $f(x) = x^2 - 2$, with $x_0 = 0$ and $x_1 = 1$, what is x_2?

7. What is x_2 if $x_0 = 1$, $x_1 = 2$, $f(x_0) = 2$, and $f(x_1) = 1.5$ in an application of the secant method?