Math 310 **Homework 1** **Due Friday, 9/11.**

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1. Section 1.1 Book Problems (Third Edition): 1, 2, 7, 9 – 11 on Pages 8 and 9.

   (1) Find the quotient q and remainder r when a is divided by b without using technology: $(i)a = 17; b = 4$, $(ii)a = 0; b = 19$, $(iii)a = -17; b = 4$.

   (2) Find the quotient q and remainder r when a is divided by b without using technology: $(i)a = -51; b = 6$, $(ii)a = 302; b = 19$, $(iii)a = 2000; b = 17$.

   (7) Prove that the square of any integer a is either of the form $3k$ or of the form $3k + 1$ for some integer k.

   (9) Prove that the cube of any integer has to be exactly one of these forms: $9k$ or $9k + 1$ or $9k + 8$ for some integer k.

   (10) Let n be a positive integer. Prove that a and c leave the same remainder when divided by n if and only if $a - c = nk$ for some integer k.

   (11) Prove the Extended Division Algorithm: Let a and b be integers with $b \neq 0$. Then there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < |b|$. NOTE: b can now be negative!

2. Section 1.2 Book Problems (Third Edition): 4, 5, 8, 19 on Pages 14 – 17.

   (4) Prove the following:

   (a) If $a|b$ and $a|c$, prove that $a|(b + c)$.

   (b) If $a|b$ and $a|c$, prove that $a|(br + ct)$ for any $r, t \in \mathbb{Z}$.

   (5) If $a|b$ and $b|a$, prove that $a = \pm b$.

   (8) Prove that $(n, n + 1) = 1$ for every integer n.

   (19) If $a|(b + c)$ and $(b, c) = 1$, prove that $(a, b) = 1 = (a, c)$.

Web Ex.1: Prove or disprove: If $a|(b + c)$, then $a|b$ or $a|c$.

Web Ex.2: If $k = abc + 1$, then prove that $(k, a) = (k, b) = (k, c) = 1$.

Web Ex.3: Prove or disprove each of the following statements.

(a) If $2 \nmid a$, then $4|(a^2 - 1)$.

(b) If $2 \nmid a$, then $8|(a^2 - 1)$.