Integration Practice

1. Let T be the region bounded by the planes $x = 0$, $y = 0$, and $z = 2$, and the surface $z = x^2 + y^2$ and lying in the quadrant $x \geq 0$, $y \geq 0$. Sketch the region and compute

$$\iiint_T x \, dxdydz.$$

2. Evaluate
$$\int_0^1 \int_0^x \int_{x^2+y^2}^2 dz\, dy\, dx\,.$$
Sketch the region T of integration and describe it.

3. Find the z-coordinate of the center of mass of T, where T is the solid tetrahedron bounded by the four planes $x = 0, y = 0, z = 0$, and $x + y + z = 1$ and $\delta(x, y) = 1$. [Hint: The mass of T is $m = \frac{1}{6}$.]

4. Find the moment of inertia of T about the y-axis, where T is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$. Assume constant density, δ.

Use the change of variable formula to calculate

$$\iint_R \cos(x+2y)\sin(x-y)\,dxdy\,,$$

over the triangular region R bounded by the lines $y=0$, $y=x$, and $x+2y=8$.

5. Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$(x, y, z) = \mathbf{T}(r, \theta, z) = (r\cos\theta, r\sin\theta, z)\,.$$

What happens to the solid box $W = [1/2, 1] \times [0, \pi] \times [0, 1]$?

6. Let $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $\mathbf{T}(u, v, w) = (2u, 2u + 3v + w, 3w)$. How does this transformation change volume?