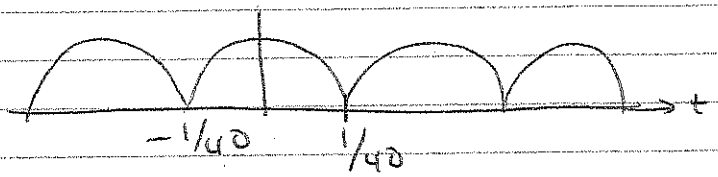
HW 8 Solns

(3) $x(t) = |\cos(20\pi t)|$ (10 points)



$T_0 = \frac{1}{20}$

$\omega_0 = \frac{2\pi}{1/20} = 40\pi$

$g(t) = \cos(20\pi t)\,\text{rect}\left(\frac{t}{1/20}\right)$

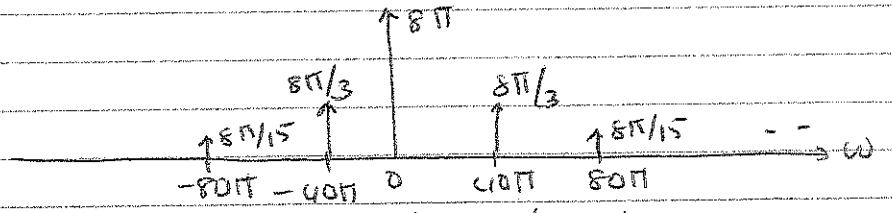
$G(\omega) = \pi[\delta(\omega-20\pi) + \delta(\omega+20\pi)] + \frac{1}{20}\,\text{sinc}\left(\frac{\omega}{40}\right)$

$= \frac{\pi}{20}\,\text{sinc}\left(\frac{\omega-20\pi}{40}\right) + \frac{\pi}{20}\,\text{sinc}\left(\frac{\omega+20\pi}{40}\right)$

$X(\omega) = \sum \omega_0 G(k\omega_0)\,\delta(\omega - k\omega_0)$

$= \sum 40\pi\left(\frac{\pi}{20}\,\text{sinc}\left(\frac{k40\pi - 20\pi}{40}\right) + \frac{\pi}{20}\,\text{sinc}\left(\frac{k40\pi + 20\pi}{40}\right)\right)\delta(\omega - 40\pi k)$

$= \sum\left[2\pi^2\,\text{sinc}\left(\frac{k40\pi - 20\pi}{40}\right) + 2\pi^2\,\text{sinc}\left(\frac{k40\pi + 20\pi}{40}\right)\right]\delta(\omega - 40\pi k)$



$k=0 \quad 2\pi^2\,\text{sinc}\left(-\frac{\pi}{2}\right) + 2\pi^2\,\text{sinc}\left(\pi/2\right)$

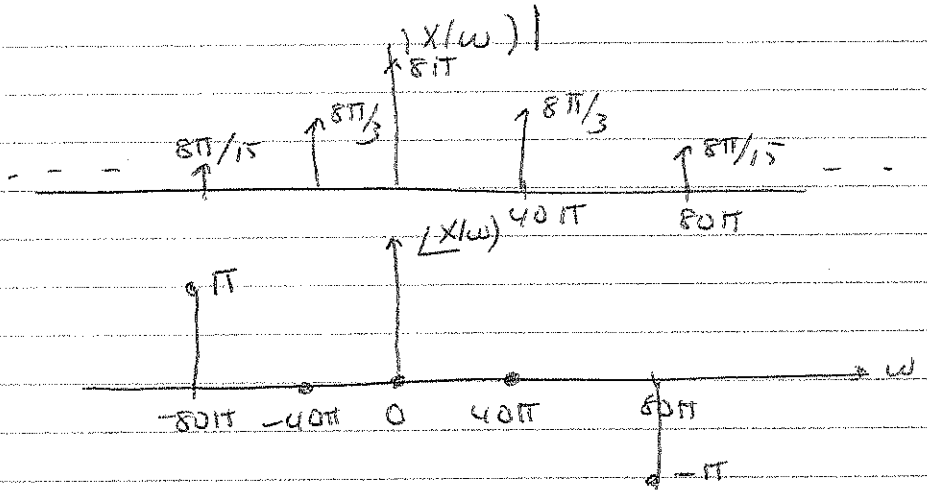
$= 2\pi^2\left(\frac{2}{\pi}\right) + 2\pi^2\left(\frac{2}{\pi}\right) = 8\pi.$

$k=1 \quad 2\pi^2\,\text{sinc}\left(\pi/2\right) + 2\pi^2\,\text{sinc}\left(\frac{3\pi}{2}\right) = 4\pi - 2\pi^2\left(\frac{2}{3\pi}\right) = 8\pi/3$

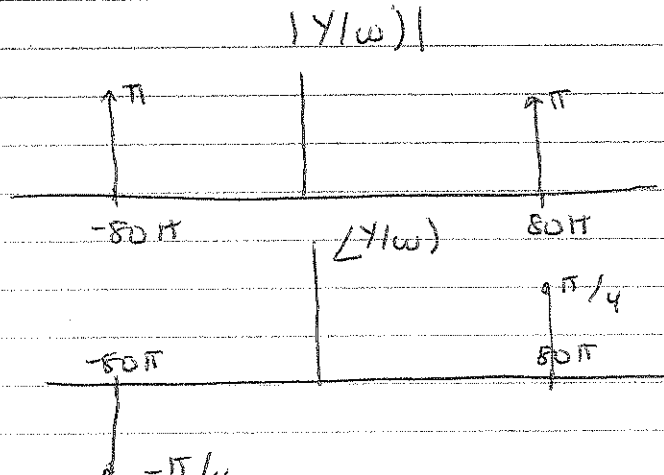
$k=2 \quad 2\pi^2\,\text{sinc}\left(\frac{3\pi}{2}\right) + 2\pi^2\,\text{sinc}\left(\frac{5\pi}{2}\right) = -\frac{4\pi}{3} + 2\pi^2\left(\frac{2}{5\pi}\right) = -\frac{8\pi}{15}$

b) $C_k = \pi \operatorname{sinc}\left(\frac{k40\pi - 20\pi}{40}\right) + \pi \operatorname{sinc}\left(\frac{k40\pi + 20\pi}{40}\right)$

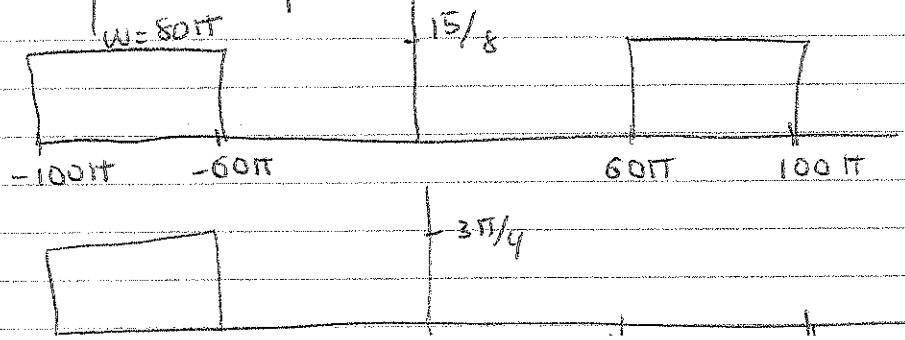
c) BPF centered at 80π with BW less than 80π.



$|H(\omega)|_{\omega=80\pi} = \frac{15}{8}$ $\quad Y(\omega) = \pi\left[e^{-j\pi/4}\delta(\omega - 80\pi) + e^{-j\pi/4}\delta(\omega + 80\pi\right.$



$\angle H(\omega) = -3\pi/4$ $\quad |H(\omega)|$



Band Pass Filter!

ECE 366 HW #8
Solutions

(2) 7.6-2 (4 points)

$$\int_{-\infty}^{\infty} \text{sinc}^2(kx)\,dx = \frac{\pi}{k}$$

if $x(t) = \text{sinc}^2(kt)$, $X(\omega) = \frac{\pi}{k}\left(\frac{\omega}{k}\right)$

and $\int |x(t)|^2 dt = \int \text{sinc}^2(kt)\,dt = \frac{1}{2\pi}\int |X(\omega)|^2 d\omega$

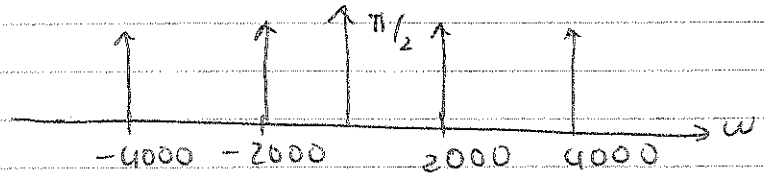
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi^2}{k^2}\,\text{rect}^2\left(\frac{\omega}{2k}\right)d\omega = \frac{1}{2\pi}\left(\frac{\pi^2}{k^2}\right)\int_{-k}^{k} \text{rect}^2\left(\frac{\omega}{2k}\right)d\omega$$

$$= \frac{1}{2\pi}\left(\frac{\pi^2}{k^2}\right)(2k) = \frac{\pi}{k}.$$

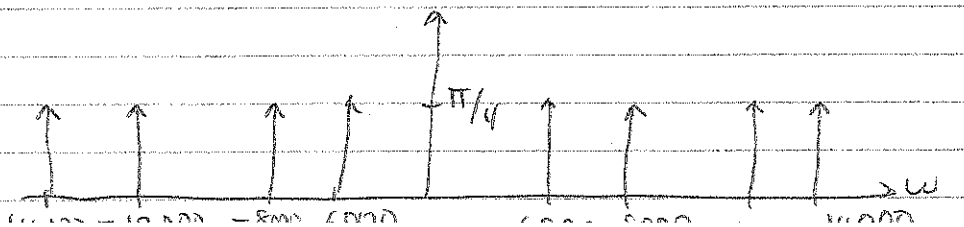
(4 points) (3) 7.7-1 (iii) $m(t) = \cos(1000t)\cos(3000t)$

$$m(t) = \frac{1}{2}\cos(2000t) + \frac{1}{2}\cos(4000t)$$

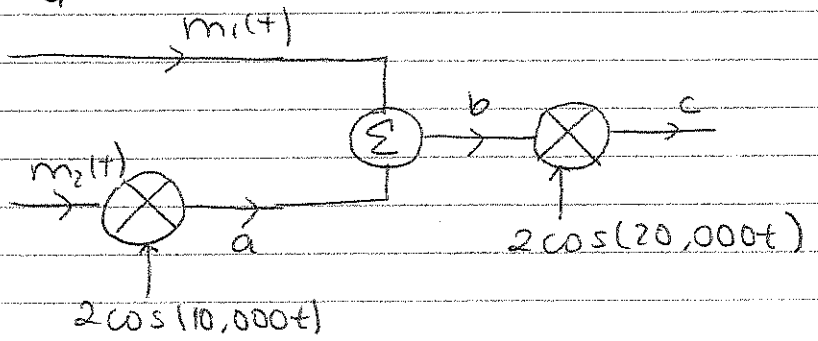
a) $M(\omega) = \frac{\pi}{2}[\delta(\omega - 2000) + \delta(\omega + 2000)] + \frac{\pi}{2}[\delta(\omega - 4000) + \delta(\omega + 4000)]$



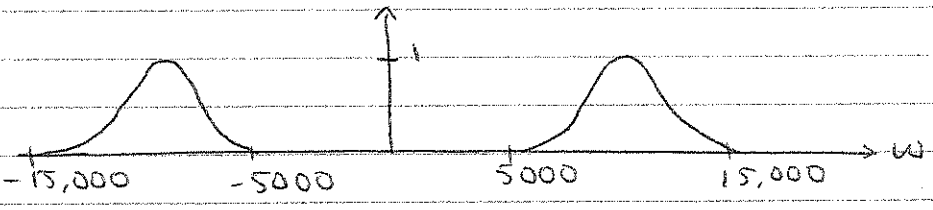
b) $m(t)\cos(10{,}000t) \leftrightarrow \frac{1}{2}M(\omega - 10{,}000) + \frac{1}{2}M(\omega + 10{,}000$



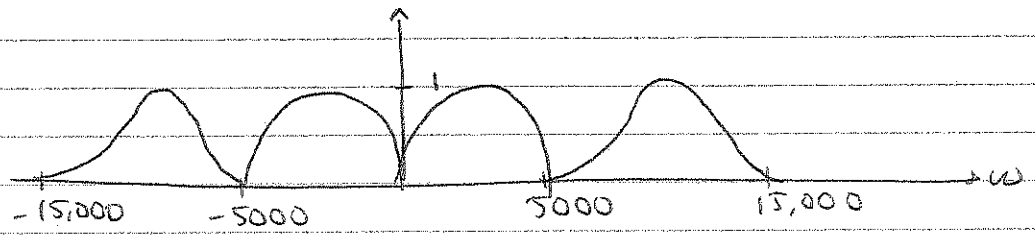
(6 points) (4) 7.7-4



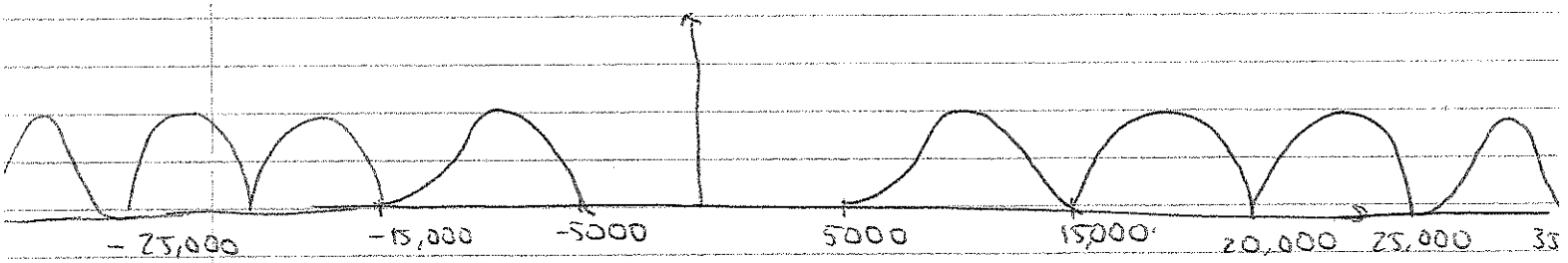
a) $A(\omega) = M_2(\omega - 10{,}000) + M_2(\omega + 10{,}000)$



$B(\omega) = A(\omega) + M_1(\omega)$



$C(\omega) = B(\omega - 20{,}000) + B(\omega + 20{,}000)$



b) BW = 35,000 − 5000 = 30,000 rad/sec

c)