1. Compute the following convolutions $x * h$ using either the integration and/or the graphical computation methods. Show all work. [10 *points*]

   (a) $x(t) = \exp(t)u(-t)$, $h(t) = -\delta(t) + 2\exp(-t)u(t)$

   (b) $x(t) = \sin(3t)u(t)$, $h(t) = \exp(-t)u(t)$

   (c) $x_1(t)$ and $x_2(t)$ in Figure P2.4-18 (a) on page 237.

   (d) $x(t) = t\,[u(t+1) - u(t-1)]$, $h(t) = u(t) + u(t-2) - u(t-4)$

   (e) $x(t) = 2u(t+2) - 2u(t-2)$, $h(t) = \exp(-|t|)\,[u(t+4) - u(t-4)]$

2. In this problem you will use MATLAB to numerically compute some convolutions. Answer all the questions and submit all the plots described below. [5 *points*]

   (a) We want to convolve the rectangular function $x(t) = u(t) - u(t-4)$ with itself using MATLAB. To do this, we sample $x(t)$ at $t = 1, 2, 3, 4$ in order to form a vector `x` with four entries, `x=[1,1,1,1]`. This can be accomplished quickly by typing

   ```
   x=ones(1,4)
   ```

   at the MATLAB prompt. Here we interpret the first entry of `x` as the value of $x(t)$ at $t = 1$, the second entry of `x` as the value of $x(t)$ at $t = 2$, etc.. Next, compute the numerical convolution of the vector `x` with itself in MATLAB by typing

   ```
   y=conv(x,x)
   ```

   at the prompt. Plot the result, and then describe/interpret each entry of the resulting vector `y` as a sample from the convolution function $(x * x)(t)$ at a particular time. That is, find times $t_1 < t_2 < \ldots$ so that the first entry of the vector `y` is equal to $(x * x)(t_1)$, the second entry of the vector `y` is equal to $(x * x)(t_2)$, etc..

   (b) Now convolve the vectors `x` and `y` from above using `conv`, and then plot the result. What is the true time duration of $x * (x * x)$, and how does it compare to what's graphed in your plot of `conv(x,y)`? Describe the plot's appearance – does it look more like a constant function, a piecewise linear function, or a quadratic function?

   (c) Now convolve a rectangular function on $[0, 1]$ with itself in the same way as for part (a). That is, represent this new function $x_1(t) = u(t) - u(t-1)$ as a vector of its values at the times $t = .25, .5, .75$, and 1, and consider it to be zero for times outside of $[0, 1]$. Use the `conv` function to plot $x_1 * x_1$. What do you have to do differently in order to make sure that your plot has the correct maximum height? Why does it make sense?

3. Consider the LTI system, T, with the input and output related by

$$y(t) = T\,[x(t)] = \int_0^t \exp(-\tau)x(t-\tau)\, d\tau.$$

Answer the following questions [5 *points*].

(a) Find the system impulse response $h(t)$ by letting $x(t) = \delta(t)$.

(b) Is this system causal? Why?

(c) Determine the system response $y(t)$ for the input $x(t) = u(t+1)$.

(d) Suppose we form a new system, T_{new}, by setting $T_{\text{new}}[x(t)] = T[x(t) - x(t-1)]$. Find the impulse response of T_{new}.

(e) Find the response of T_{new} to the input $x(t) = u(t+1)$.

ECE 360 HW#4
Solutions

(1) a) $x(t) = e^{t}u(-t)$
$h(t) = -\delta(t) + 2e^{-t}u(t)$

$y(t) = x(t) * h(t) = -e^{t}u(-t) + \underbrace{2e^{-t}u(t) * e^{t}u(-t)}$

$$2\int_{-\infty}^{\infty} e^{\tau}u(-\tau)e^{-(t-\tau)}u(t-\tau)\,d\tau$$

$$= 2\int_{-\infty}^{0} e^{2\tau}e^{-t}u(t-\tau)\,d\tau = 2e^{-t}\int_{-\infty}^{0} e^{2\tau}\underbrace{u(t-\tau)}_{\begin{cases}1, & t \geq \tau\\ 0, & t < \tau\end{cases}}\,d\tau$$

if $t \geq 0 \rightarrow 2e^{-t}\int_{-\infty}^{0} e^{2\tau}\,d\tau = 2e^{-t}\left.\frac{e^{2\tau}}{2}\right|_{-\infty}^{0}$
$= 2e^{-t}\left[\frac{1}{2}\right] = e^{-t}$

if $t < 0 \rightarrow 2e^{-t}\int_{-\infty}^{t} e^{2\tau}\,d\tau = e^{-t}\left.e^{2\tau}\right|_{-\infty}^{t}$
$= e^{-t} \cdot e^{2t} = e^{t}$

$y(t) = e^{t}u(-t) + e^{-t}u(t) = e^{-|t|}$ //

b) $x(t) = \sin(3t)u(t) \quad h(t) = e^{-t}u(t)$

$$\int_{-\infty}^{\infty} \sin(3\tau)u(\tau)e^{-(t-\tau)}u(t-\tau)\,d\tau$$

$$= \int_{0}^{\infty} \sin(3\tau)e^{-t}e^{\tau}u(t-\tau)\,d\tau$$

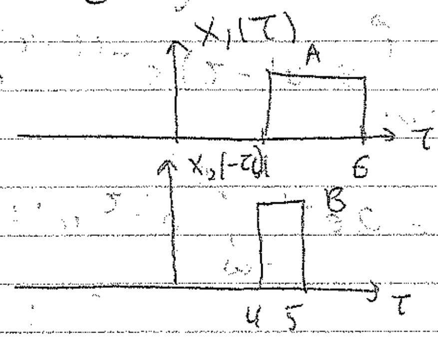
$= e^{-t}\int^{t} \sin(3\tau)e^{\tau}d\tau \rightarrow$

$$e^{-t}\,\frac{e^{\tau}}{10}\left(\sin(3\tau) - 3\cos(3\tau)\right)\Big|_0^t$$

$$= e^{-t}\left[\frac{e^{t}}{10}\left(\sin(3t) - 3\cos(3t)\right) - \frac{1}{10}(-3)\right]$$

$$= \left[\frac{1}{10}\sin(3t) - \frac{3}{10}\cos(3t) + \frac{3}{10}e^{-t}\right]u(t)$$

c)



if $t \geq 2$ $y(t) = 0$

if $t < -1$ $y(t) = 0$

$-1 \leq t < 0$ partial overlap

$$\int_4^{5+t} AB\,d\tau = AB(t+1)$$

$0 \leq t \leq 1$ → full overlap

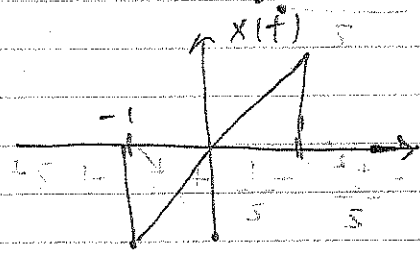
$$\int_{4+t}^{5+t} AB\,d\tau = AB$$

if $1 \leq t < 2$ → partial overlap $\int_{4+t}^{6} AB\,d\tau$

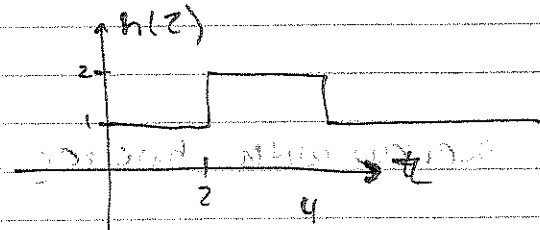
$= AB(2-t)$

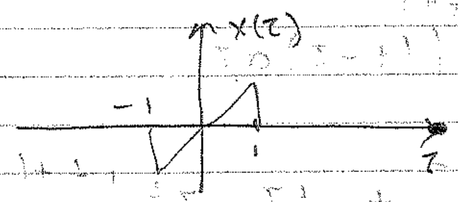
$$y(t) = \begin{cases} 0 & t < -1 \\ ABt + AB & -1 \leq t < 0 \\ AB & 0 \leq t < 1 \\ 2AB - ABt & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

(1d) $x(t) = t\,[u(t+1) - u(t-1)]$

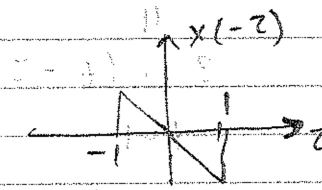


$h(t)$



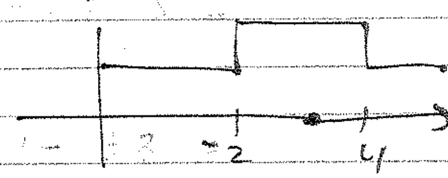


→

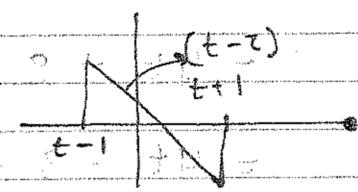


if $t \le -1$ no overlap $y(t) = 0$

$-1 < t \le 1$ partial overlap



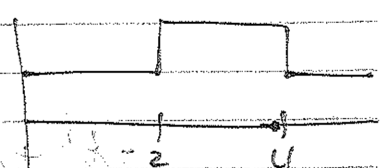
$$\int_0^{t+1} (t-\tau)\,d\tau$$



$$t\tau - \frac{\tau^2}{2}\Bigg|_0^{t+1}$$

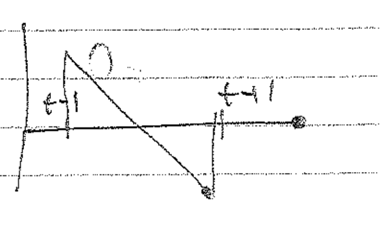
$$= t(t+1) - \frac{(t+1)^2}{2} = t^2 + t - \frac{t^2+2t+1}{2}$$

$$y(t) = \frac{t^2}{2} - \frac{1}{2} \quad -1 < t \le 1$$



$1 < t < 3$ overlap with both 1 & 2

$$\int_{t-1}^{2} (t-\tau)\,d\tau + 2\int_{2}^{t+1} (t-\tau)\,d\tau$$



$$= t\tau - \frac{\tau^2}{2}\Bigg|^{2} + 2\left(t\tau - \frac{\tau^2}{2}\right)\Bigg|^{t+1}$$

$$2t - 2 - t(t-1) - \frac{(t-1)^2}{2} + 2t(t+1) - (t+1)^2$$

$$-4t + 4$$

$$= -\cancel{2t} + 2 - \cancel{t^2} + \cancel{t} - \cancel{\frac{t^2}{2}} - \frac{1}{2} + \cancel{t} + \cancel{2t^2} + \cancel{2t} - \cancel{t^2} - 1 - \cancel{2t}$$

$$= -\frac{t^2}{2} + \frac{1}{2}$$

$3 \le t < 5$ overlap with two regions

$$2\int_{t-1}^{4} (t-\tau)\,d\tau + \int_{4}^{t+1} (t-\tau)\,d\tau$$

$$= 2\left[t\tau - \frac{\tau^2}{2} \Big|_{t-1}^{4} \right] + t\tau - \frac{\tau^2}{2} \Big|_{4}^{t+1}$$

$$= 8t - 16 - 2t(t-1) + (t-1)^2 + t(t+1) - \frac{(t+1)^2}{2}$$

$$-4t + 8$$

$$= 4t - 8 - \cancel{2t^2} + \cancel{2t} + \cancel{t^2} - \cancel{2t} + 1 + \cancel{t^2} + \cancel{t} - \cancel{\frac{t^2}{2}} - \frac{1}{2} - \cancel{t}$$

$$= -\frac{t^2}{2} + 4t - \frac{15}{2}$$

$5 \le t$ full overlap $\quad \int_{t-1}^{t+1} (t-\tau)\,d\tau = t\tau - \frac{\tau^2}{2} \Big|_{t-1}^{t+1}$

$$= t(t+1) - \frac{(t+1)^2}{2} - t(t-1) + \frac{(t-1)^2}{2}$$

$$= \cancel{t^2} + \cancel{t} - \cancel{t^2} + \cancel{t} - \cancel{\frac{t^2}{2}} - \cancel{\frac{1}{2}} - \cancel{t} + \cancel{\frac{t^2}{2}} - \cancel{t} + \cancel{\frac{1}{2}}$$

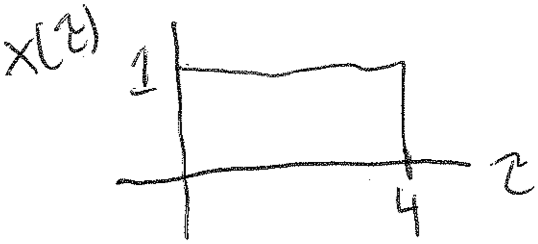
$$= 0$$

(i.e.) on your own

m- code for problem 2

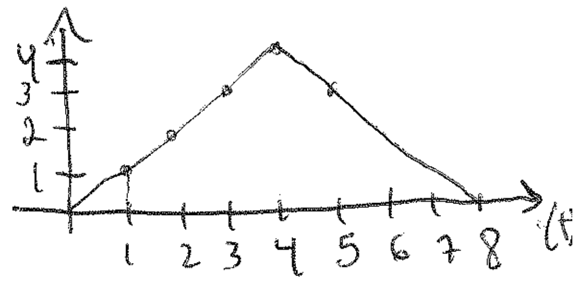

```
% Clear the memory
clear;

% Sample u(t) - u(t-3) at t = 0, 1, 2, 3.
x = ones(1,4);

% Compute the convolution of x with itself, and then plot the result
y = conv(x,x);

figure;
plot(y);

% Compute the convolution of x with y.
z = conv(x,y);

figure;
plot(z);

% Now compute the convolution of x1 = u(t) - u(t-1) with itself, and plot
% the result.

x1 = ones(1,4);
y1 = .25*conv(x1,x1);  % We multiply by .25 because our spacing between
    samples from x1 is .25!

figure;
plot(.25*[1:7],y1);
```

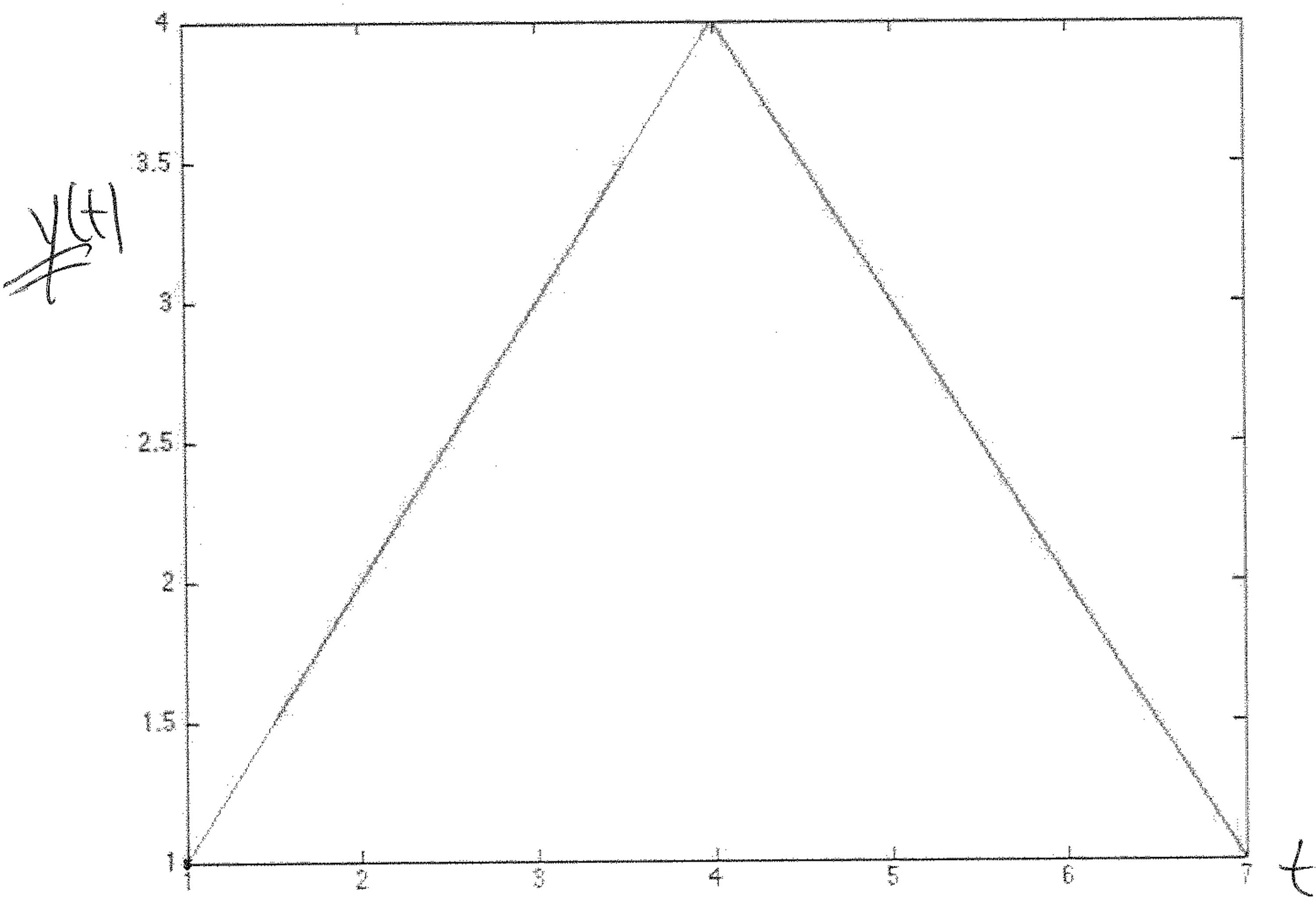
plot for #2 (a): $x(t) = u(t) - u(t-4)$



;

$\underline{x * x}$:

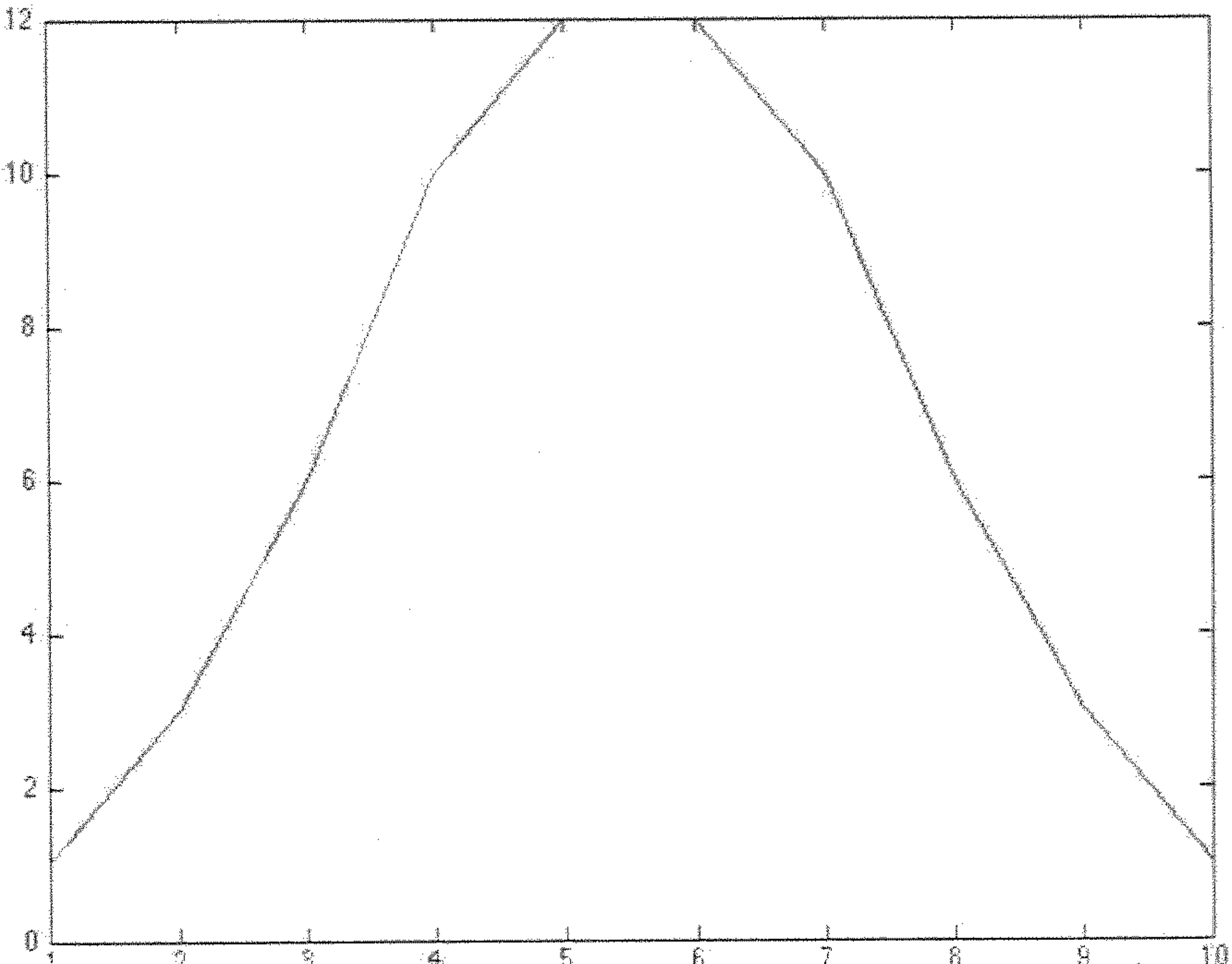




$y = [1, 2, 3, 4, 3, 2, 1]$ ← samples at $t = 1, 2, 3, 4, 5, 6, 7$

$= [x*x(1),\ x*x(2),\ x*x(3),\ x*x(4),\ x*x(5),\ x*x(6),\ x*x(7)]$

#2. (b) plot: It is more like a quadratic function! It only looks piecewise linear because of how it's plotted...



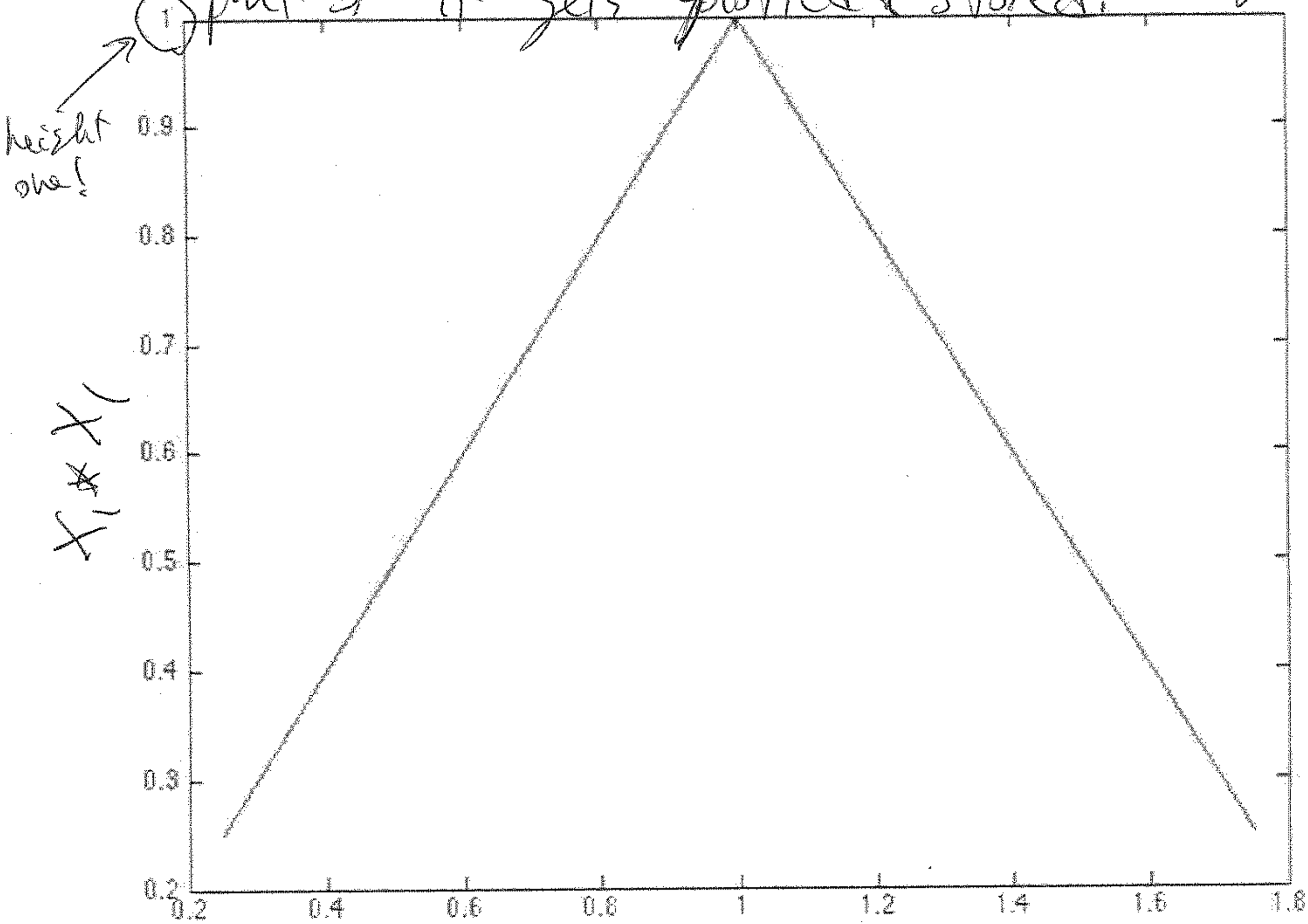
Only part of the total time duration of (x**x)**x is plotted!

The true time duration = time duration of x
+ time duration of y
$= 4 + 8 = \boxed{12}$

plot for #2 (c) : $x_1(t) = u(t) - u(t-1)$.

→ this is the plot of $\underline{1/4}\cdot \mathrm{conv}(x_1, x_2)$

→ correct time duration is $\boxed{2}$. Only part of it gets plotted & stored.



→ Multiplying by 1/4 gets the plot correct because our time spacing is 1/4!

(3) $y(t) = \int_0^t e^{-\tau} x(t-\tau)\, d\tau$

a) $h(t) = \int_0^t e^{-\tau} \delta(t-\tau)\, d\tau$

$= \int_0^t e^{-t} \delta(t-\tau)\, d\tau$

$= e^{-t} \int_0^t \delta(t-\tau)\, d\tau$

$= e^{-t} u(t)$

b) Yes, $h(t) = 0$ for $t < 0$

c) $y(t) = u(t+1) * e^{-t} u(t)$

$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t-\tau+1)\, d\tau$

$= \int_0^{\infty} e^{-\tau}\, u(t-\tau+1)\, d\tau$, $t-\tau+1 \geq 0$

$t+1 \geq \tau$

$= \int_0^{t+1} e^{-\tau}\, d\tau = -e^{-\tau}\Big|_0^{t+1}$ $\quad t+1 \geq 0$

$t \geq -1$

$= -e^{-(t+1)} + 1$

$= \left(1 - e^{-(t+1)}\right) u(t+1)$

d) $h(t) - h(t+1) = h_{new}(t)$

$h_{new}(t) = h(t) - h(t-1)$

$= e^{-t} u(t) - e^{-(t-1)} u(t-1)$

e) $y_{new}(t) = h_{new}(t) * u(t+1)$

$= h(t) * u(t+1) - h(t-1) * u(t+1)$ } use shift