ECE 366
HW 8
Solutions

1) a) $x[n] = (0.5)^n u[n-1] \quad h[n] = 2^n u[2-n]$

$$y[n] = \sum_{k=-\infty} 2^k u[2-k] \, (0.5)^{n-k} u[n-k-1]$$

$$(0.5)^n \sum_{k=-\infty}^{2} (4)^k u[n-k-1]$$

if $n-1 \geq 2$, $n \geq 3$ $\quad (0.5)^n \sum_{k=-\infty}^{2} (4)^k = (0.5)^n \frac{(-64)}{(1-4)}$

$$= \frac{64}{3}(0.5)^n$$

if $n-1 < 2$ $\quad (0.5)^n \sum_{k=-\infty}^{n-1} (4)^k = (0.5)^n \frac{(-4^n)}{-3} = \frac{(2)^n}{3}$

$$y[n] = \begin{cases} \frac{(2)^n}{3} & n < 3 \\ \frac{64}{3}(0.5)^n & n \geq 3 \end{cases}$$

b) $x[n] = u[n] \quad h[n] = \delta[n] - 2\delta[n-1]$

$y[n] = u[n] - 2u[n-1]$

c)

n	-4	-3	-2	-1	0	1	2	3	4	5
$x[k]$	0	1	1	1	1	1	1	1	0	0
$h[-k]$	0	1	1	1	1	1	1	1	0	0

$y[0] = 7 \quad y[1] = 6 \quad y[2] = 5 \quad y[3] = 4 \quad y[4] = 3$
$y[5] = 2 \quad y[6] = 1 \quad y[7] = 0 \quad y[n] = 0 \quad n \geq 7$
$y[-1] = 6 \quad y[-2] = 5 \quad y[-3] = 4 \quad y[-4] = 3 \quad y[-5] = 2$
$y[-6] = 1 \quad y[-7] = 0 \quad y[n] = 0 \quad n \leq -7$

d) $x[n] = e^{-n} u[n+1] \quad h[n] = (-2)^n u[n-1]$

$$y[n] = \sum (-2)^k u[k-1] \, e^{-(n-k)} u[n-k+1]$$

$$= e^{-n} \sum_{k=1}^{\infty} (-2)^k (e)^k u[n-k+1] \qquad \begin{matrix}(n+1 \geq 1) \\ n \geq 0\end{matrix}$$

$$= e^{-n} \sum_{k=1}^{n+1} (-2e)^k = e^{-n} \left[\frac{-2e - (-2e)^{n+2}}{1+2e} \right] u[n]$$

2) $h[n] = \delta[n] + (1/3)^n u[n-1]$

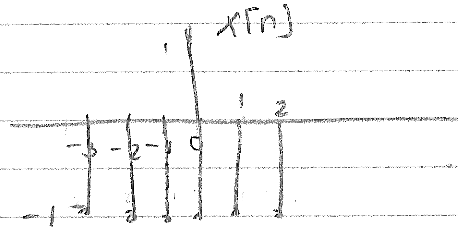
a) causal since $h[n] = 0 \quad n < 0$

$$\sum |h[n]| = 1 + \sum_{n=1}^{\infty} (1/3)^n$$

$$= 1 + \frac{1/3}{1 - 1/3} = 3/2 < \infty$$

stable

b) $u[n-3] + u[n+3]$



c) $y[n] = x[n] * h[n]$

$y[n] = x[n] + x[n] * (1/3)^n u[n-1]$

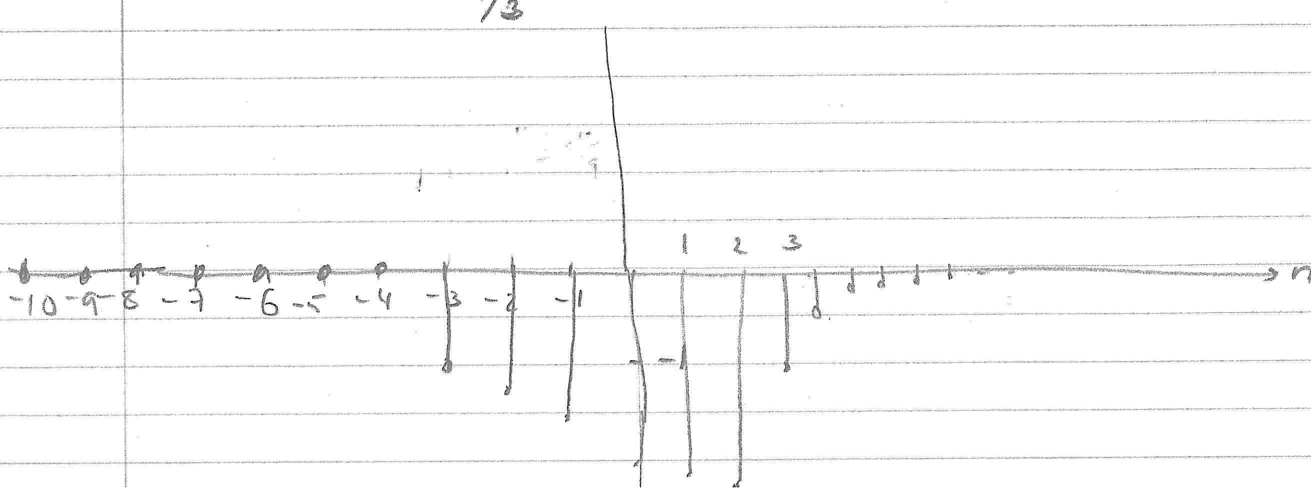
	-4	-3	-2	-1	0	1	2	3	4	
$h[k]$	0	0	0	0	0	1/3	1/9	1/27	1/81	- -
$x[-k]$	0	0	-1	-1	-1	-1	-1	-1	0	- -

$y_1[0] = -1/3 + -1/9 + -1/27$ $\quad y_1[n] = 0 \quad n \leq -3$

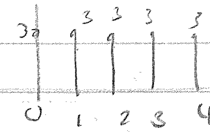
$y_1[-2] = -1/3 \quad y[-1] = -1/3 + \frac{-1}{9}$

$$y_1[n] = \sum_{k=1}^{n+3} (-1/3)^k \quad , n \geq -2$$

$$y_1[n] = \frac{-1/3 - (-1/3)^{n+4}}{4/3} \qquad y[n] = x[n] + y_1[n]$$



(3) a) $x[n] = 3(u[n] - u[n-5])$



$$X(z) = 3 + 3z^{-1} + 3z^{-2} + 3z^{-3} + 3z^{-4}$$

ROC: whole z-plane except $z = 0$.

DTFT exists since ROC includes the unit circle

b) $x[n] = 2^{n+1} u[n-1] + e^{n-1} u[n]$

$$= (4)(2^{n-1}) u[n-1] + e^{-1} e^{n} u[n]$$

$$X(z) = 4 z^{-1} \left(\frac{z}{z-2} \right) + e^{-1} \left(\frac{z}{z-e} \right)$$

$$= \frac{4}{z-2} + \frac{e^{-1} z}{z-e}$$

$|z| > 2$ and $|z| > e$

ROC: $|z| > e$

DTFT does not exist

c) $x[n] = n 2^{n} u[n-1]$

$$= n 2^{n} u[n] \quad (\text{since at } n = 0 \; x[n] = 0)$$

$$2^{n} u[n] \longleftrightarrow \frac{z}{z-2} \qquad n 2^{n} u[n] \longleftrightarrow -z \frac{d}{dz} \frac{z}{z-2}$$

$$= -z \left(\frac{1}{z-2} - \frac{z}{(z-2)^2} \right) = -z \left(\frac{-2}{(z-2)^2} \right) = \frac{2z}{(z-2)^2}$$

ROC: $|z| > 2$

DTFT does not exist

d) $\left[2^{-n} \cos\left(\frac{\pi}{3} n \right) \right] u[n-1] = \left[2^{-n} \cos\left(\frac{\pi}{3} n \right) \right] u[n] - \delta[n]$

$$X(z) = \frac{z\left(z - \frac{1}{2} \cos(\pi/3)\right)}{z^2 - \cos\left(\frac{\pi}{3}\right) z + 1/4} - 1$$

$$= \frac{z\left(z - 1/4\right)}{\left(z^2 - \frac{1}{2} z + 1/4\right)} - 1 = \frac{\cancel{z^2} - 1/4\, z - \cancel{z^2} + 1/2\, z - 1/4}{(z - 1/2)^2}$$

$$= \frac{1/4\, z - 1/4}{(z - 1/2)^2}$$

ROC: $|z| > 1/2$, DTFT does not exist

(4) 5.1-5 a, g, k

a) $\frac{z(z-4)}{z^2-5z+6} = \frac{z(z-4)}{(z-2)(z-3)}$

$\frac{X(z)}{z} = \frac{k_1}{z-2} + \frac{k_2}{z-3}$ $\quad k_1 = \frac{-2}{-1} = 2$

$k_2 = \frac{-1}{1} = -1$

$x[n] = 2(2)^n u[n] - (3)^n u[n]$

g) $\frac{z(1.4z+0.08)}{(z-0.2)(z-0.8)^2}$

$\frac{X(z)}{z} = \frac{k_1}{z-0.2} + \frac{k_2}{(z-0.8)^2} + \frac{k_3}{z-0.8}$

$k_1 = \frac{(1.4)(0.2)+0.08}{(0.36)} = 1$

$k_2 = \frac{(1.4)(0.8)+0.08}{(0.6)} = \frac{1.12+0.08}{0.6} = \frac{1.2}{0.6} = 2$

$\lim_{z\to\infty} X(z) = 1 + k_3 = 0 \quad k_3 = -1$

$X(z) = (0.2)^n u[n] - (0.8)^n u[n] + 2.5n(0.8)^n u[n]$

k) $\frac{z(3.83z+11.34)}{(z-2)(z^2-5z+25)} \rightarrow \frac{X(z)}{z} = \frac{k_1}{z-2} + \frac{k_2}{z-2.5-2.5\sqrt{3}j} + \frac{k_2^*}{z-2.5+2.5\sqrt{3}j}$

$\frac{5 \pm \sqrt{25-100}}{2} = \frac{5}{2} \pm \frac{5\sqrt{3}}{2} j$

$k_1 = \frac{(3.83)(2)+11.34}{(19)} = 1$

$k_2 = \frac{(3.83)(2.5+j2.5\sqrt{3})+11.34}{(0.5+j2.5\sqrt{3})(5\sqrt{3}j)}$

$x[n] = \left[(2)^n u[n] + \sqrt{2}(5)^n \cos\left(\frac{\pi}{3}n - \frac{3\pi}{4}\right)\right] u[n]$

(5) 5.2-9 c, e, i

c) $\sum_{k=0}^{4} z^{-2k} = 1 + z^{-2} + z^{-4} + z^{-6} + z^{-8}$

Finite sequence

$x[n] = \{1, 0, 1, 0, 1, 0, 1, 0, 1$

(18)

e) $\frac{z^2}{z^4 - 1} =$ $z^2 \;\big)\; z^4 - 1$ ... $z^{-2} + z^{-6} + z^{-10}$

$z^2 - z^{-2}$

z^{-2}

$z^{-2} - z^{-6}$

z^{-6}

(15)

i) $\frac{z}{z - 1.1} \rightarrow (1.1)^n u[n]$ (16)

(6) $y[n+2] - 3y[n+1] + 2y[n] = x[n+1]$

$y[-1] = 2 \quad y[-2] = 3 \quad x[n] = (3)^n u[n]$

$\rightarrow y[n] - 3y[n-1] + 2y[n-2] = x[n-1]$

$Y(z) - 3(z^{-1} Y(z) + y[-1]) + 2(z^{-2} Y(z) + z^{-1} y[-1] + y[-2])$

$= z^{-1}(X(z) + \overset{0}{\cancel{x[-1]}})$

$\rightarrow Y(z) - \frac{3}{z} Y(z) - \cancel{6} + \frac{2}{z^2} Y(z) + \frac{4}{z} + \cancel{6} = \frac{1}{\cancel{z}} \frac{\cancel{z}}{z-3}$

$Y(z)\left(1 - \frac{3}{z} + \frac{2}{z^2}\right) + \frac{4}{z} = \frac{1}{z-3}$

$Y(z)\left(\frac{z^2 - 3z + 2}{z^2}\right) = \frac{1}{z-3} - \frac{4}{z}$

$$Y(z)\left(\frac{z^2-3z+2}{z^2}\right) = \frac{z-4(z-3)}{z(z-3)}$$

$$Y(z) = \frac{-3z+12}{\cancel{z}(z-3)}\left(\frac{z^{\cancel{2}}}{(z-1)(z-2)}\right)$$

$$Y(z) = \frac{z(-3z+12)}{(z-3)(z-1)(z-2)}$$

$$\frac{Y(z)}{z} = \frac{k_1}{z-3} + \frac{k_2}{z-1} + \frac{k_3}{z-2}$$

$$k_1 = \frac{3}{(2)(1)} = 3/2, \quad k_2 = \frac{9}{(-2)(-1)} = \frac{9}{2}, \quad k_3 = \frac{6}{(-1)} = -6$$

$$y[n] = 3/2\,(3)^n u[n] + \frac{9}{2}u[n] - 6\,(2)^n u[n] \,//$$

(7) 5-3-18

a) $H(z) = \dfrac{z}{(z+0.2)(z-0.8)}$

$x[n] = e^{(n+1)} u[n] = e\, e^n u[n] \longleftrightarrow X(z) = \dfrac{ez}{z-e}$

$Y(z) = X(z)\,H(z) = \dfrac{e\, z^2}{(z+0.2)(z-0.8)(z-e)}$

$\dfrac{Y(z)}{z} = \dfrac{k_1}{z+0.2} + \dfrac{k_2}{z-0.8} + \dfrac{k_3}{z-e}$

$k_1 = \dfrac{e(-0.2)}{(-1)(-0.2-e)} = -0.1863$

$k_2 = \dfrac{e(0.8)}{(1)(0.8-e)} = -1.1336$

$k_3 = \dfrac{e^2}{(e+0.2)(e-0.8)} = 4.8571$

$y[n] = -0.1863(-0.2)^n u[n] - 1.1336(0.8)^n u[n]$
$\quad + 4.8571 (e)^n u[n]$

b) $\dfrac{Y(z)}{X(z)} = \dfrac{z}{z^2 - 0.6z - 0.16}$

$zX(z) = z^2 Y(z) - 0.6 z Y(z) - 0.16 Y(z)$

$x[n+1] = y[n+2] - 0.6\, y[n+1] - 0.16\, y[n]$

(8) 5.5-5

$y[n+1] - 0.5\, y[n] = x[n+1] + 0.8\, x[n]$

a) $zY(z) - 0.5 Y(z) = zX(z) + 0.8 X(z)$

$Y(z)(z-0.5) = X(z)(z+0.8)$

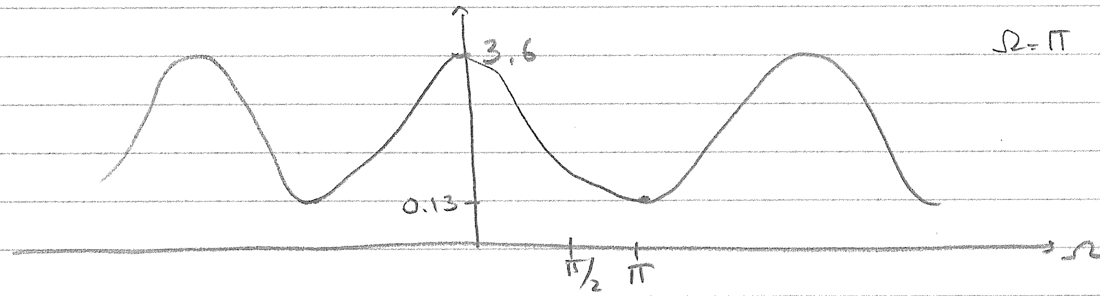
$H(z) = \dfrac{z+0.8}{z-0.5}$

$$H(e^{j\Omega}) = \frac{e^{j\Omega}+0.8}{e^{j\Omega}-0.5} = \frac{(\cos\Omega+0.8)+j\sin\Omega}{(\cos\Omega-0.5)+j\sin(\Omega)}$$

$$|H(e^{j\Omega})| = \frac{\sqrt{1.64+1.6\cos(\Omega)}}{\sqrt{1.25-\cos(\Omega)}}$$

$\Omega=0$ $\quad \frac{1.8}{0.5} = 3.6$

$\Omega=\pi$ $\quad \frac{0.2}{1.5}$ $\quad 0.133$



$$\angle H(e^{j\Omega}) = \tan^{-1}\frac{\sin(\Omega)}{\cos(\Omega)+0.8} - \tan^{-1}\frac{\sin(\Omega)}{\cos(\Omega)-0.5}$$