1. Determine whether the following signals are periodic or not. If periodic, find the period:

   (a) $x[n] = \exp(\mathrm{i}0.25n/\pi)$
   (b) $x[n] = \exp(\mathrm{i}2\pi n/.25)$
   (c) $x[n] = \sin(3.5\pi n/7)$
   (d) $x[n] = \exp(\mathrm{i}2\pi n/.25) + \cos(\pi n)$

2. 3.4-13 on page 330.

3. Determine whether the following discrete-time systems are linear, time-invariant, memoryless, causal and stable.

   (a) $y[n] = \frac{x[n]}{x[n+3]}$
   (b) $y[n] = \sin\left(\frac{\pi}{2}(n+1)\right)x[n]$
   (c) $y[n] = \sum_{k=-n}^{n}\left(x[k+a]\right)^{1.1}$, a is an integer.

4. 3.7-1 on page 331.

5. 3.8-2 on page 331.

HW 10 - Soln's

#1. a.) $x[n] = e^{in/4\pi}$

$$\Omega_0 = \frac{1}{4\pi} = \left(\frac{1}{8\pi^2}\right)2\pi \Rightarrow \boxed{\text{Not periodic}}$$

b.) $x[n] = e^{in2\pi/.25} = e^{i8\pi n}$

$$\Omega_0 = 8\pi = \left(\frac{4}{1}\right)2\pi \Rightarrow \boxed{\text{Periodic, } N_0 = 1}$$

c.) $x[n] = \sin(3.5\pi n/7) = \sin(\pi n/2)$

$$\Omega_0 = \pi/2 = \left(\frac{1}{4}\right)2\pi \Rightarrow \boxed{\text{Periodic, } N_0 = 4}$$

d.) $x[n] = \exp\left(i\frac{2\pi n}{.25}\right) + \cos(\pi n)$

$$= \exp(i8\pi n) + \cos(\pi n)$$

$$\Omega_1 = 8\pi = 4(2\pi) \text{ \& } \Omega_2 = \left(\frac{1}{2}\right)2\pi$$

$$\Rightarrow \boxed{\text{Periodic, } N_0 = 2}$$

#2. 3.4-13: $y[n] = r[n]\,x[n] = n\,u[n]\,x[n]$

(a) Let $x[n] = 1$. Then, $|y[n]| = |x[n] \cdot n| = n$ (if $n \geq 0$) $\to \infty$ as $n \to \infty$. $\boxed{\text{Not Stable}}$

(b) If $y_1[n] = r[n] \cdot x_1[n]$ & $y_2[n] = r[n] x_2[n]$, $a, b \in \mathbb{R}$,

then $y[n] = r[n]\left(a x_1[n] + b x_2[n]\right) = a\, r[n]\, x_1[n] + b\, r[n]\, x_2[n]$

$= a y_1[n] + b y_2[n]$

$\boxed{\text{linear!}}$

(c) It is $\boxed{\text{memoryless}}$, $y[n] = r[n]\, x[n]$ only depends on $x[n]$.

(d) It is $\boxed{\text{causal}}$ also since $y[n]$ only depends on $x[n]$.

(e) $x[n-m] \longrightarrow r[n]\, x[n-m]$, but

$y[n-m] = r[n-m]\, x[n-m] \neq r[n]\, x[n-m]$!

$\boxed{\text{Not time invariant!}}$

#3. (a) $y[n] = x[n] / x[n+3]$

Not linear: $\dfrac{a x_1[n] + b x_2[n]}{a x_1[n+3] + b x_2[n+3]} \neq a \dfrac{x_1[n]}{x_1[n+3]} + b \dfrac{x_2[n]}{x_2[n+3]}$

in general

Time invariant: $x[n-m] \longrightarrow \dfrac{x[n-m]}{x[n-m+3]} = y[n-m]$

#3 (a) cont'd:

Not Memoryless: $y[0]$ depends on $y[0]$ & $y[3]$

Not Causal:

Not Stable: $x[n] = u[n] - u[n-5]$ has

$$y[3] = \frac{x[3]}{x[6]} = \frac{1}{0} = \infty$$

(b) $y[n] = \sin\left(\frac{\pi}{2}(n+1)\right) \cdot x[n]$

Linear: $a x_1[n] + b x_2[n] \longrightarrow \sin\left(\frac{\pi}{2}(n+1)\right)\left[a x_1[n] + b x_2[n]\right]$

$$= a\left(\sin\left(\frac{\pi}{2}(n+1)\right) x_1[n]\right) + b\left(\sin\left(\frac{\pi}{2}(n+1)\right) x_2[n]\right) = a y_1[n] + b y_2[n]$$

Time Varying: $x[n-m] \longrightarrow \sin\left(\frac{\pi}{2}(n+1)\right) \cdot x[n-m]$

$$\neq$$

$$y[n-m] = \sin\left(\frac{\pi}{2}(n-m+1)\right) x[n-m]$$

Memoryless: Yes (only depends on current input)

Causal: Yes

Stable: If $x[n] \leq M$ then $|y[n]| \leq \left|\sin\left(\frac{\pi}{2}(n+1)\right)\right| M$

$$\leq M$$

#3. (c) $y[n] = \sum_{k=-n}^{n} \left(x[k+a]\right)^{1.1}$

Not Linear: $a x_1[n] + b x_2[n] \longrightarrow \sum_{k=-n}^{n} \left(a x_1[k+a] + b x_2[k+a]\right)^{1.1}$

$$\neq$$

$$a \sum_{k=-n}^{n} \left(x_1[k+a]\right)^{1.1} + b \sum_{k=-n}^{n} \left(x_2[k+a]\right)^{1.1}$$

in general

Time Varying: $y[n-m] = \sum_{k=-n+m}^{n-m} \left(x[k+a]\right)^{1.1}$ while $\neq$

$x[n-m] \longrightarrow \sum_{k=-n}^{n} \left(x[k-m+a]\right)^{1.1}$

Memoryless: No
Causal: No
Stable: No $\rightsquigarrow \sum_{k=-n}^{n} \left(x(k+a)\right)^{1.1} = (2n+1) M^{1.1}$

When $x[n] = M$, $(2n+1) M^{1.1} \to \infty$ as $n \to \infty$

#4. 3.7-1: (a) $h[n+1] + 2h[n] = \delta[n]$; $h[n] = 0$ for $n<0$ works.

$n=-1$: $h[0] + 2h[-1] = \delta[-1] = 0 \Rightarrow h[0] = 0$
$n=0$: $h[1] + 2h[0] = \delta[0] = 1 \Rightarrow h[1] = 1$
$n=1$: $h[2] + 2h[1] = \delta[2] = 0 \Rightarrow h[2] = -2$
$n=2$: $h[3] + 2h[2] = \delta[3] = 0 \Rightarrow h[3] = 4$

$\vdots$

$$\boxed{h[n] = (-2)^{n-1} u[n-1]}$$

#4. (b) $h[n] + 2h[n-1] = \delta[n]$; $h[n] = 0$ for $n<0$ works!

$n=0$: $h[0] + 2h[-1] = 1 \Rightarrow h[0] = 1$

$n=1$: $h[1] + 2h[0] = 0 \Rightarrow h[1] = -2$

$n=2$: $h[2] + 2 \cdot h[1] = 0 \Rightarrow h[2] = 4$

$\vdots$

$$h[n] = (-2)^n u[n]$$

#5. 3.8-2: $x[n] = 3^{n-1} u[n+2]$ and

$$h[n] = \frac{1}{2}\left[\delta[n-2] - (-2)^{n+1}\right] u[n-3]$$
$$= -(-2)^n u[n-3]$$

k:	-4	-3	-2	-1	0	1	2	3	4	5	6	7	...
$x[k]$:	0	0	3^{-3}	3^{-2}	3^{-1}	3^0	3^1	3^2	3^3	3^4	3^5	3^6	...
$h[-k]$:	$-(-2)^4$	$-(-2)^3$	0	0	0	0	0	0	0	0	0	0	

$y[0] = 0$ & $\boxed{y[n] = 0 \ \forall n < 0}$

$$y[1] = -(-2)^3 \cdot 3^{-3} = -\left(\frac{-2}{3}\right)^3$$

$$y[2] = -(-2)^4 \cdot 3^{-3} + -(-2)^3 \cdot 3^{-2} = -3\left[\left(\frac{-2}{3}\right)^4 + \left(\frac{-2}{3}\right)^3\right]$$

$$y[3] = -(-2)^5 3^{-3} + -(-2)^4 3^{-2} + -(-2)^3 3^{-1} = -3^2\left[\left(\frac{-2}{3}\right)^5 + \left(\frac{-2}{3}\right)^4 + \left(\frac{-2}{3}\right)^3\right]$$

$\vdots$

$$y[n] = -3^{n-1} \cdot \sum_{k=0}^{n-1} \left(\frac{-2}{3}\right)^{3+k} = -3^{n-1}\left(\frac{-2}{3}\right)^3 \frac{1-\left(\frac{-2}{3}\right)^n}{1+\frac{2}{3}}$$

for $n \geq 1$