ECE 366, Exam 2 — March 30, 2015

Name: ______________________________

Instructions:

- No electronic devices that can access the web, exchange text, other messages, etc., can be used
- You must show appropriate legible work and justify your answer to receive full credit.
- There are 30 possible points. Point values for each problem are as indicated.
- Check and make sure there are four total pages including the cover page, when you begin the exam.
- You must read and sign the honor code below before your test will be graded.

Good Luck!

Problem	Score	Out of
1		5
2		5
3		7
4		5
5		8
Total		30

ACADEMIC HONOR CODE

As a student and citizen of the Michigan State University Community I pledge to not lie, cheat, or steal in my academic endeavors.

SIGNED: ______________________________

1. **Transfer Functions**: Suppose that an LTI system has the transfer function $H(s) = \frac{s}{(s+1)(s+2)}$. What is the output of the system when $x(t) = 2\exp(-3t) + 5$? Show all work. [5 *points*]

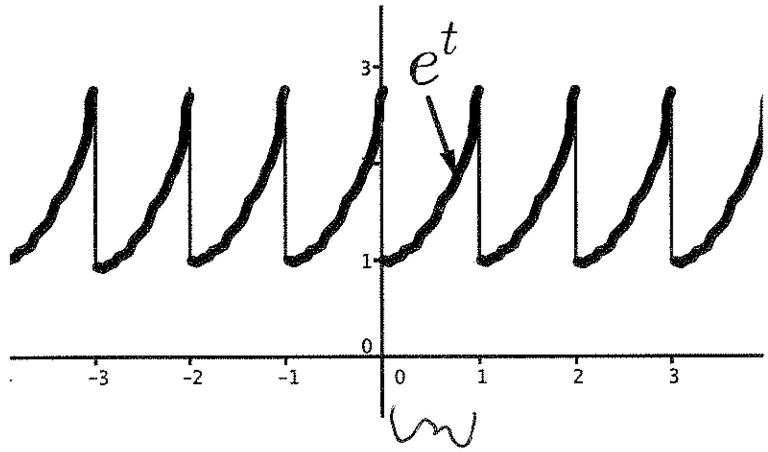
$$5 = 5 \cdot e^{0t} \xrightarrow{\text{LTI}} 5 \cdot \underbrace{H(0)}_{=0} \cdot e^{0t} = 0$$

and

$$2e^{-3t} \xrightarrow{\text{LTI}} 2 \cdot H(-3)\, e^{-3t} = 2\left(\frac{-3}{-2 \cdot -1}\right) e^{-3t} = -3e^{-3t}$$

$$\boxed{x(t) \xrightarrow{\text{LTI}} -3e^{-3t}}$$

2. **Fourier Series:** Find the Fourier Series coefficients, C_k, for the periodic exponential function below. [5 *points*]:



$T_0 = 1$, so $\omega_0 = 2\pi$

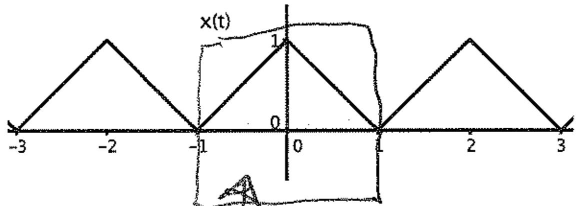
$$C_k = \frac{1}{1}\int_0^1 e^t e^{-ik2\pi t}\,dt$$

$$= \int_0^1 e^{(1-ik2\pi)t}\,dt$$

$$= \frac{1}{1-ik2\pi}\left(e^{(1-ik2\pi)t} \Big|_0^1 \right)$$

$$= \boxed{\frac{e^{1-ik2\pi} - 1}{1-ik2\pi}}$$

3. **Fourier Transforms and Filtering:** Consider the following periodic tent function, $x(t)$:



(a) Compute the Fourier Transform of $x(t)$. Show all work, and reference the notes/book as necessary. [4 *points*]

$$g(t) = \text{tent}\left(\frac{t}{2}\right) = \Delta\left(\frac{t}{2}\right)$$

book p. 702: $g(t) = \Delta\left(\frac{t}{2}\right) \xleftrightarrow{\mathcal{F}} \text{sinc}\left(\frac{\omega}{2}\right) = \hat{g}(\omega)$

$$\text{So, } \hat{x}(\omega) = \omega_0 \sum_{k=-\infty}^{\infty} \text{sinc}^2\left(\frac{k\omega_0}{2}\right) \delta(\omega - k\omega_0)$$

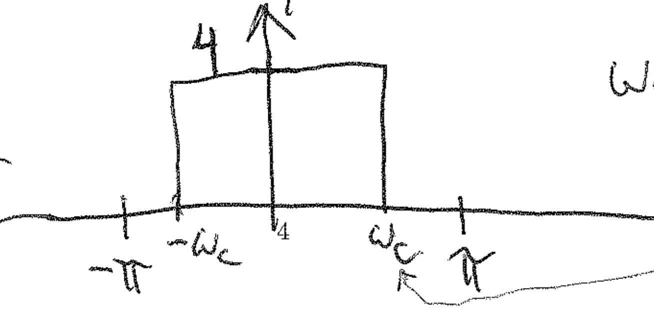
where $T = 2 \Rightarrow \omega_0 = \frac{2\pi}{2} = \pi$. Thus

$$\boxed{\hat{x}(\omega) = \pi \sum_{k=-\infty}^{\infty} \text{sinc}^2\left(\frac{k\pi}{2}\right) \delta(\omega - k\pi)}$$

(b) Suppose your boss asks you to design a Low Pass Filter (LPF) whose output when given $x(t)$ is $y(t) = 2$. Graph the frequency response, $\hat{h}(\omega)$, of the LPF you would design in order to accomplish this task. Label all important points. [3 *points*]

$\hat{x}(0) = \pi$ and $y(t) = 2 \xleftrightarrow{\mathcal{F}} 4\pi\delta(\omega)$

$\hat{h}(\omega)$:



works $\hat{h}(0) = 4$, $\omega_c < \pi$

4. **Properties of the Fourier Transform:** Suppose that the Fourier transform of the signal $x(t)$ is $\hat{x}(\omega) = \exp(-\omega^2)$. Calculate the Fourier transform of the modulated derivative of $x(t)$,

$$z(t) = \frac{dx}{dt}\sin(10\pi t).$$

Show all work. Use at most one property of the Fourier Transform at a time. [5 *points*]

$$\mathcal{F}[z] = \frac{1}{2\pi}\mathcal{F}\left[\frac{dx}{dt}\right] * \mathcal{F}\left[\sin(10\pi t)\right]$$

$$\rightsquigarrow \mathcal{F}\left[\frac{dx}{dt}\right] = i\omega e^{-\omega^2}$$

$$\rightsquigarrow \sin(10\pi t) = \frac{e^{i10\pi t} - e^{-10i\pi t}}{2i} \overset{\mathcal{F}}{\longleftrightarrow} i\pi\left[\delta(\omega+10\pi) - \delta(\omega-\right.$$

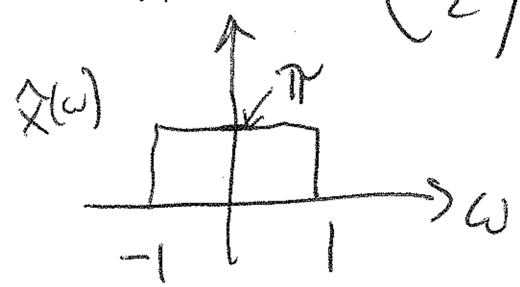
So,

$$\hat{z}(\omega) = -\frac{1}{2}\left[(\omega+10\pi)e^{-(\omega+10\pi)^2} - (\omega-10\pi)e^{-(\omega-10\pi)^2}\right]$$

5. **Properties of the Fourier Transform:** Use Parseval's Theorem to compute the energy of $x(t) = \sin(t)/t$. Show all work. [8 *points*]

$$\text{sinc}(t) = \frac{\sin t}{t}, \quad \frac{1}{\pi}\,\text{sinc}(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{rect}\left(\frac{\omega}{2}\right)$$

$$\text{so, } \hat{X}(\omega) = \pi\,\text{rect}\left(\frac{\omega}{2}\right)$$



Thus, the energy of $x(t)$ is

$$\int_{-\infty}^{\infty} \text{sinc}^2 t \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\pi\,\text{rect}\left(\frac{\omega}{2}\right)\right)^2 d\omega$$

Parseval's

$$= \frac{1}{2\pi}\pi^2 \int_{-1}^{1} d\omega = \boxed{\pi}$$