1. For the systems described by the following input-output relationships, determine whether the systems are (*i*) linear, (*ii*) time-invariant, (*iii*) instantaneous, (*iv*) causal, (*v*) stable, and (*vi*) invertible. Please show your work to receive full credit. [12 *points*].

   (a) $y(t) = \int_{-\infty}^{t/2} x(\tau)\ d\tau$

   (b) $y(t) = 3\ x\,(3t + 3)$

   (c) $y(t) = x(t)\ t\ u(t)$

   (d) $y(t) = \frac{d}{dt}\,[e^{-t}\ x(t)]$

2. Simplify the following expressions as much as possible: [5 *points*].

   (a) $\frac{\sin t}{t^2+2}\ \delta(t)$

   (b) $\frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+4}\delta(1-t)$

   (c) $\int_{-\infty}^{\infty} \sin(\pi t)\ \delta(2t-3)\ dt$

   (d) $\int_{-\infty}^{t-1} e^{-\tau}\ \delta(\tau+2)\ d\tau$

   (e) $\int_{-\infty}^{\infty} \sin\,(3t-3)\ \ \delta(2t+4)\ dt$

3. A linear system has the input-output pairs given below:

   - If $x_1(t) = u(t) - u(t-1)$, then $y_1(t) = t\,[u(t) - u(t-1)]$
   - If $x_2(t) = u(t-1) - u(t-3)$, then $y_2(t) = u(t) - (t-1)u(t-1) + (t-2)u(t-3)$
   - If $x_3(t) = u(t-1) - u(t-2)$, then $y_3(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-3)$

   Answer the following questions and explain your answers. [5 *points*].

   (a) Sketch each one of the inputs and outputs.

   (b) Could this system be causal?

   (c) Could this system be time invariant?

   (d) Could this system by memoryless?

   (e) What is the output of this system for $x(t) = u(t) + u(t-1) - 2u(t-2)$