Any of the following exercises are fair game for your final oral exam. I suggest that you write up your solutions neatly in your own handwriting to consult during that exam!

Additional Reminder: You have an option of doing a project instead of doing the homework... so keep that in mind (and check the syllabus) if that sounds more appealing.

Basic Probability Review Problems

1. Write down the density for the result of a fair 6-sided die role using Diracs.
2. Show that if X and Y are independent random numbers, then $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$.
3. If $a \in \mathbb{R}$ show that $\mathrm{Var}[aX] = a^2 \mathrm{Var}[X]$.
4. Let $\{X_j\}_{j\in\mathbb{N}} \subset \mathbb{R}$ be a sequence of independent random numbers with $\mathbb{E}[X_j] = \mu$ and bounded variance $\mathbb{Var}[X_j] \leq \sigma^2$ for all $j \in \mathbb{N}$. Use the Chebyshev Inequality to argue that
$$\mathbb{P}\left[\lim_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n} X_j = \mu\right] = 1.$$
5. Let $X \in \mathbb{R}^+$ be a non-negative random variable. Show that $\mathbb{E}[X] = \int_0^\infty \mathbb{P}[X > t]\, dt$.[1]
6. Let X be a random variable and $p \in (0, \infty)$. Use the result of problem 5 together with a change of variable to show that
$$\mathbb{E}[|X|^p] = \int_0^\infty pt^{p-1}\, \mathbb{P}[X > t]\, dt.$$
7. Show that any two distinct entries of $X \sim \mathcal{N}(0, I_n)$, X_j and X_k with $k \neq j$, are independent.
8. Do homework exercise 2.3.1 on page 44 of the notes (`https://math.msu.edu/~iwenmark/Notes_Fall2020_Iwen_Classes.pdf`).
9. Do homework exercise 2.7.2 on page 65 of the notes (`https://math.msu.edu/~iwenmark/Notes_Fall2020_Iwen_Classes.pdf`).

[1] Your solution might benefit from writing X in terms of the random indicator function $\mathbb{1}_{\{t<X\}} := \begin{cases} 1 \text{ if } t < X \\ 0 \text{ else} \end{cases}$.