CMSE 823, **Computational Practice Problems** **Dec. 8, 2016**

Name: ______________________________

1. **BY HAND - Arnoldi:** Let $A \in \mathbb{R}^{4\times 4}$ and $\mathbf{b} \in \mathbb{C}^4$ be

$$A = \begin{pmatrix} -1 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{7}{3} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ -3 \\ 4 \end{pmatrix}.$$

Do one Arnoldi iteration beginning with $\mathbf{b}$ in order to generate a the factorization $AQ_1 = Q_2 H_1$ where $Q_1 \in \mathbb{C}^{4\times 1}$ consists of a single unit norm column, $Q_2 \in \mathbb{C}^{4\times 2}$ has two orthonormal columns, and $H_1 \in \mathbb{C}^{2\times 1}$. **CHECK YOU ANSWER WITH A NEIGHBOR!**

2. **BY HAND - GMRES:** Approximate the solution to $A\mathbf{x} = \mathbf{b}$ using your answer to the first question by: (i) solving for

$$\beta := \arg\min_{\alpha \in \mathbb{C}} \|AQ_1\alpha - \mathbf{b}\|_2,$$

and then (ii) using β to approximate the solution $\mathbf{x} \in \mathbb{C}^4$.

3. **BY HAND - Arnoldi:** Let $A \in \mathbb{R}^{4\times 4}$ and $\mathbf{b} \in \mathbb{C}^4$ be

$$A = \begin{pmatrix} -1 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{7}{3} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ -3 \\ 4 \end{pmatrix}.$$

Do two Arnoldi iterations beginning with $\mathbf{b}$ in order to generate a the factorization $AQ_2 = Q_3 H_2$ where $Q_2 \in \mathbb{C}^{4\times 2}$ has two orthonormal columns, $Q_3 \in \mathbb{C}^{4\times 3}$ has three orthonormal columns, and $H_2 \in \mathbb{C}^{3\times 2}$ is in Hessenberg form. **CHECK YOU ANSWER WITH A NEIGHBOR!**

4. **BY HAND - Arnoldi:** Let $A \in \mathbb{R}^{4\times 4}$ and $\mathbf{b} \in \mathbb{C}^4$ be

$$A = \begin{pmatrix} -1 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{7}{3} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ -3 \\ 4 \end{pmatrix}.$$

(a) Write down all eigenvalues of A.

(b) Use the upper 2×2 submatrix of H_2 from the last problem in order to approximate two eigenvalues and eigenvectors of A.

(c) Did you find any real eigenvalues and eigenvectors of A this way?