Math 309, Section 4 – Exam 1 Review

Midterm Exam 1 will be given in class on Wednesday, October 2nd. It will have around 6 questions. The material of the midterm covers the material of Section 1.1 – 1.5, 3.1, 3.2.

What to study:

0. Notation. You should be familiar with the notation and terms that the book uses routinely, including: the coefficient matrix and augmented matrix of a linear system, the vector spaces $\mathbb{R}^n$, $\mathbf{P}_n$, $\mathbb{R}^{m\times n}$ and $\mathcal{C}[a,b]$. You should, of course, know how to multiply matrices and the properties of matrix multiplication, including understanding the formulas:

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}, \qquad (AB)^{-1} = B^{-1}A^{-1}, \qquad (AB)^T = B^T A^T.$$

1. Definitions. You should be able to give *precise* definitions of the following terms:

Caution: For each term, find the precise definition in the textbook. Copy it over several times to learn it. If the definition in class uses different language from the textbook, you can use either. Make sure you are mathematically consistent (think about Math 299 when writing the definitions).

Be sure to include all hypotheses (e.g. "If A is an $n \times n$ matrix") and all logical quantifiers (e.g. "for all", "for some", "there exists", etc.).

linear system	solution set	elementary row operations
consistent system	row echelon form	reduced row echelon form
Identity matrix I_n	inverse of a matrix	transpose of a matrix
elementary matrices (3 types)	singular matrix	non-singular matrix
Vector space (full definition)	subspace	Null space of a matrix
Span$\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$	spanning set for a subspace	

2. Theorems. There were a few theorems in the class. Make sure you understand what Theorems 1.5.1, 1.5.2, Corollary 1.5.3, 3.2.1 and other theorems from your notes. You don't need to memorize proofs of these particular theorems. But you need to know how to apply them to the problems.

3. Calculations. You should know how to

- Find the solution set of a linear system. Identify lead and free variables.
- Transform matrices to row echelon form.
- Multiply matrices.
- Work with elementary matrices: write down an elementary matrix corresponding to a row operation.

- Use row reduction to check whether a given matrix is singular or non-singular.
- Find the inverse of a matrix (by the formula for 2×2, and by row reduction for larger matrices).
- Finding a null subspace of a matrix (like in Chapter 3.2 example 9 or in class).
- Determine if a given set of vectors is a spanning set of $\mathbb{R}^n$ (make a matrix, transform into echelon form, make conclusions from it).

4. Proofs. There will be several proofs on the exam. One will be a short proof based on the axioms of a vector space, like Lemmas 1–12 on the handout. You will need to derive something directly from axioms, without using lemmas. The list of axioms will be included on the midterm sheet.

There may be also proof-based questions, such as true/false questions, or applying theorems from the class.

Preparing for Exam 1: sample problems

Below is the list of problems of various difficulty, like the ones that can be on the exam.

- Check that you can do all homework problems.
- Section 1.2: 4, 5, 6, 8, 9, 13, 14
- Section 1.3: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15, 16.
- Section 1.4: 2, 5, 8, 9, 12, 14, 16, 22, 23, 24.
- Section 1.5: 1, 2, 3, 4, 6, 7, 9, 10, 15, 16, 17, 19.
- Section 3.1: 7, 8, 9, 10.
- Section 3.2: 1, 2, 4, 5, 11, 12, 13.
- End of Chapter 1, Chapter test A: 1, 2, 3, 4, 5, 6, 7, 8. Chapter test B: 1, 3, 5a.