

Math 828, Homework 2

Due September 20

1. Exercise 1.2.5.
2. Exercise 1.2.9.
3. Exercise 1.2.6.
4. A subset of \mathbb{R} is called *perfect* if it is closed and does not have isolated points. Show that a non-empty perfect set is uncountable. Hint: suppose that $E = \{q_1, q_2, \dots\}$ is perfect. Hint: construct a sequence of intervals $[a_1, b_1] \supset [a_2, b_2] \supset \dots$ such that $[a_n, b_n] \cap E \neq \emptyset$, but $\{q_1, \dots, q_n\} \cap [a_n, b_n] = \emptyset$.
5. Show that $[0, 1)$ cannot be represented as a union of closed disjoint intervals. Hint: show that, if true, then the set of endpoints of these intervals is perfect.
6. Exercise 1.2.14
7. Exercise 1.2.15
8. Exercise 1.2.17
9. Exercise 1.2.19