Useful algebraic identities with examples of use

- $a^2 - b^2 = (a - b)(a + b)$

  $x^2 - 25 = (x - 5)(x + 5)$

  $(\sqrt{x^2 + 5} + 3)(\sqrt{x^2 + 5} - 3) = (\sqrt{x^2 + 5})^2 - 3^2 = (x^2 + 5) - 9$

  $= x^2 - 4 = (x - 2)(x + 2)$

- $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + a^2 b^{n-3} + ab^{n-2} + b^{n-1})$

  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

  $x^3 - 1 = (x - 1)(x^2 + x + 1)$

  $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

  $x^3 + 1 = (x + 1)(x^2 - x + 1)$

  $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

  $a^4 - b^4 = (a - b)(a^3 + a^2 b + ab^2 + b^3)$

  $x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1)$

  $a^4 - b^4 = (a + b)(a^3 - a^2 b + ab^2 - b^3)$

  $x^4 - 1 = (x + 1)(x^3 - x^2 + x - 1)$

- $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$

  $x^4 - 81 = (x - 3)(x + 3)(x^2 + 9)$

- $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

  $x^2 - 2x + 1 = (x - 1)^2$

- $(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$ and $(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$