MTH 234 Quiz 1 *Solutions*
May 23, 2016 (20 points total)

For problems 1 - 4: Let $\vec{v} =< 3, 0, -1 >$, $\vec{u} =< 1, -2, 3 >$, $\vec{w} =< 9, 3, -1 >$, $P_0 = (2, 0, 1)$

1. (2 points) Which of the above vectors are orthogonal?

Solution. $\vec{v} \bullet \vec{u} = 0$ and $\vec{u} \bullet \vec{w} = 0$, so $\vec{v} \perp \vec{u}$ and $\vec{u} \perp \vec{w}$. However, $\vec{v} \bullet \vec{w} = 28$, so $\vec{v} \not\perp \vec{w}$.

2. (2 point) Compute $\vec{v} \times \vec{w}$.

Solution.

$$\vec{v} \times \vec{w} = \det \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 0 & -1 \\ 9 & 3 & -1 \end{pmatrix} = 3\hat{\imath} - 6\hat{\jmath} + 9\hat{k}.$$

3. (2 points) Compute the area of the triangle generated by $\vec{u}$ and $\vec{v}$.

Solution. Since the vectors are orthogonal, the area is just one-half of the product of their magnitudes:

$$\text{Area} = \frac{1}{2}\sqrt{10}\sqrt{14} = \sqrt{35}.$$

Alternatively, you could compute a cross product, and compute one-half of its magnitude:

$$\frac{1}{2}\left|\vec{u} \times \vec{v}\right| = \frac{1}{2}\left|\det \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ 3 & 0 & -1 \end{pmatrix}\right| = \frac{1}{2}\left|2\hat{\imath} + 10\hat{\jmath} + 6\hat{k}\right| = \frac{1}{2}\sqrt{2^2 + 10^2 + 6^2} = \sqrt{35}$$

4. (4 points) Give the equation of the plane containing P_0 and the vectors $\vec{u}$ and $\vec{w}$.

Solution.

$$\vec{u} \times \vec{w} = \det \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ 9 & 3 & -1 \end{pmatrix} = -7\hat{\imath} + 28\hat{\jmath} + 21\hat{k}.$$

So the equation of the plane is given by

$$-7(x - 2) + 28y + 21(z - 1) = 0.$$

5. (3 points) Find the volume of the parallelepiped generated by the vectors $< 1, 0, 1 >$, $< 0, 2, 0 >$, and $< 2, -1, 3 >$.

Solution.

$$\det \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 1 \\ 2 & -1 & 3 \end{pmatrix} = 2(1 \cdot 3 - 2 \cdot 1) = 2.$$

6. (3 points) Find the distance between the point $(1, -2, 5)$ and the plane $3y - 4z + 1 = 0$.

Solution. Note that $(0, 1, 1)$ is a point in the plane. (Any other point in the plane will also work.) Then a vector from the plane to $(1, -2, 5)$ is given by $< 1, -3, 4 >$. The distance is then given by taking the absolute value of the dot product with the unit normal vector

$$\text{dist} = \left| < 1, -3, 4 > \bullet \frac{< 0, 3, -4 >}{5} \right| = \frac{25}{5} = 5.$$

7. (4 points) Let L be the line connecting the points $(1, 3, -2)$ and $(-1, 0, 1)$ and let P be the plane $3x + y - 4z = 7$. Where do L and P intersect?

Solution.
A parametric equation of the line connecting the points is given by:

$$L(t) = (1 - 2t, 3 - 3t, 3t - 2)$$

So that $L(0) = (1, 3, -2)$ and $L(1) = (-1, 0, 1)$. We then solve for the value of t for which $L(t)$ intersects the plane:

$$3(1 - 2t) + (3 - 3t) - 4(3t - 2) = 7$$

$$\iff \quad 21t = 7 \quad \iff \quad t = \frac{1}{3}$$

So the point on the plane is

$$L\left(\frac{1}{3}\right) = \left(\frac{1}{3}, 2, -1\right).$$