MTH 234 Exam 2 Name ____________________________

June 20, 2016 (75 points total)

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1. (8 points) Express the volume under the plane $z = 4 + 3y - x$ and above the region bounded by $x + y = 1$ and $x^2 + y = 1$ as a double integral, and evaluate the integral.

2. (6 points) Evaluate $\iint_H \left(\sin(3x^2 + 3y^2) + 2\right)\ dA$ where H is the half disk bounded by $x^2 + y^2 = 36$ and the x-axis, where $y \geq 0$.

3. (8 points) Express the volume between the paraboloids $y = 3x^2 + 3z^2$ and $y = 10 - x^2 - z^2$ as a triple intergral, and evaluate the integral.

4. (6 points) Express the volume $V = \iiint_E \, dV$ three ways:

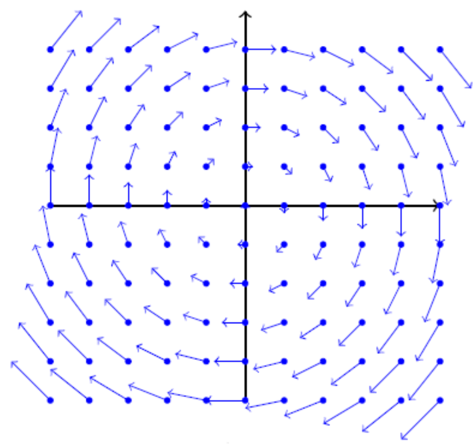
a) $dV = dydzdx$
b) $dV = dzdydx$
c) $dV = dxdzdy$

Where E is the solid bounded by the surface $16 - y - x^2 - 16z^2 = 0$ and the plane $y = 0$.

5. (9 points) Compute the gradient vector field associated to the following functions

$$f = \sqrt{x^2 - 2y^2} \qquad g = xe^{xy^2} \qquad h = x\ln(1+y^2)$$

6. (6 points) Given the following diagram of a vector field **F**, determine if the following integrals are *positive, negative, or zero*:



$$\int_{C_1} \mathbf{F} \cdot dr \qquad\qquad \int_{C_2} \mathbf{F} \cdot dr \qquad\qquad \int_{C_2} \mathbf{F} \cdot dr$$

where
C_1 is the vertical line from $(-3,-2)$ to $(-3,2)$
C_2 is the horizonal line from $(5,2)$ to $(-3,2)$
and C_3 is the circle with radius 1 centered at the origin with clockwise rotaton.

7. (8 points) Compute the surface area of the part of the paraboloid $z = 1 - x^2 - y^2$ above the xz-plane and above the xy-plane (i.e. $z \geq 0$ and $y \geq 0$.)

8. (6 points) Is the vector field $\langle y^2z^3, 2xyz^3, 3xy^2z^2\rangle$ conservative? If so, find a potential.

9. (8 points) Compute the line integral of the vector field $\mathbf{F} = \langle P(x,y), Q(x,y) \rangle$ over the boundary of the triangle T whose vertices are (0,0), (1,1) and (0,1) with counter-clockwise orientation, where

$$P(x,y) = x \int_0^y e^{-\tau^2}\, d\tau \quad \text{and} \quad Q(x,y) = \frac{1}{2}(1+x)^2 e^{-y^2}$$

10. (10 points) Compute the work done by the force field $\mathbf{F} = \left\langle x\sqrt{x^2+y^2}, 2y\sqrt{x^2+y^2} \right\rangle$ on a particle when pushing the particle one time around the unit circle $x^2+y^2=1$ in the counter-clockwise direction.